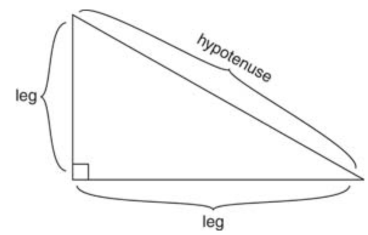
TRIGONOMETRY REVIEW

Trigonometry deals with the study of triangles. The following is a review of facts that you should have learned in previous courses.

Definitions:

Use the acronym SOHCAHTOA to remember what parts of the right triangle determine the function's value. The acronym indicates the following relations:



$$\sin(\theta) = \frac{opposite}{hypotenuse}$$
$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$
$$\tan(\theta) = \frac{opposite}{adjacent}$$

alongside these functions we also have:

$$\csc(\theta) = \frac{1}{\sin(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)} \quad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Trigonometric Identities and Properties:

Expanded Definitions
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Identities
We have three main trigonometric identities:

(1) $\sin^2(\theta) + \cos^2(\theta) = 1$
(2) $\tan^2(\theta) + 1 = \sec^2(\theta)$
Note: This comes from dividing (1) by $\cos^2(\theta)$
(3) $1 + \cot^2(\theta) = \csc(\theta)$
Note: This comes from dividing (1) by $\sin^2(\theta)$

Even and Odd Functions
An *even function* is a function such that $f(-x) = f(x)$. An *odd function* is a function such that $f(-x) = -f(x)$.

$\cos(-\theta) = \cos(\theta)$ (i.e cosine is an even function)
$\sin(-\theta) = -\sin(\theta)$ (i.e. sine is an odd function)
$\tan(-\theta) = -\tan(\theta)$ (i.e. tangent is an odd function)

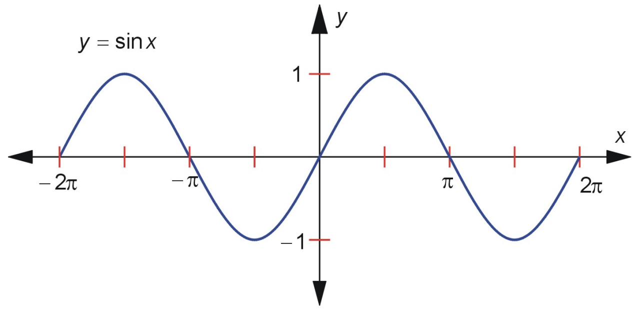
Graphs and Periods

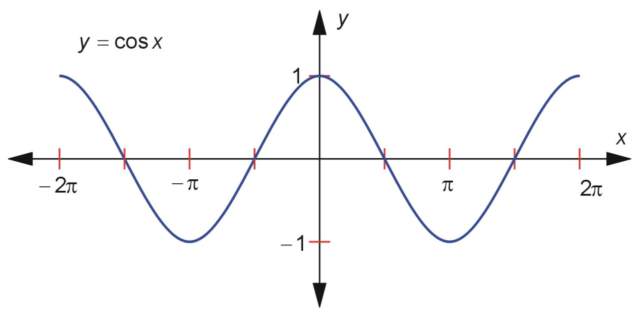
On the right are the graphs for the three main trigonometric functions. From the graphs we can easily see the following properties:

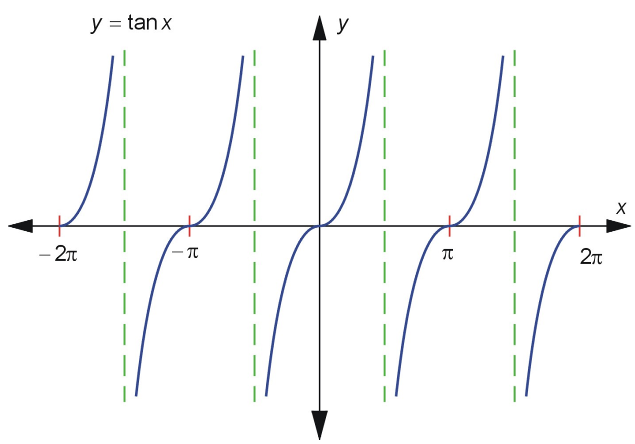
(i) Sine has range $[-1, 1]$
(ii) Cosine has range $[-1, 1]$
(iii) Tangent has range $(-\infty, \infty)$

A function f is called *periodic* if there is an $a > 0$ such that $f(x + a) = f(x)$ for all x in the domain of f. The smallest such a is called the *period* of f. (i.e. the graph repeats itself after a units)

(iv) Sine has period 2π
$\Rightarrow \sin(\theta + 2\pi) = \sin(\theta)$
(v) Cosine has period 2π
$\Rightarrow \cos(\theta + 2\pi) = \cos(\theta)$
(vi) Tangent has period π
$\Rightarrow \tan(\theta + \pi) = \tan(\theta)$



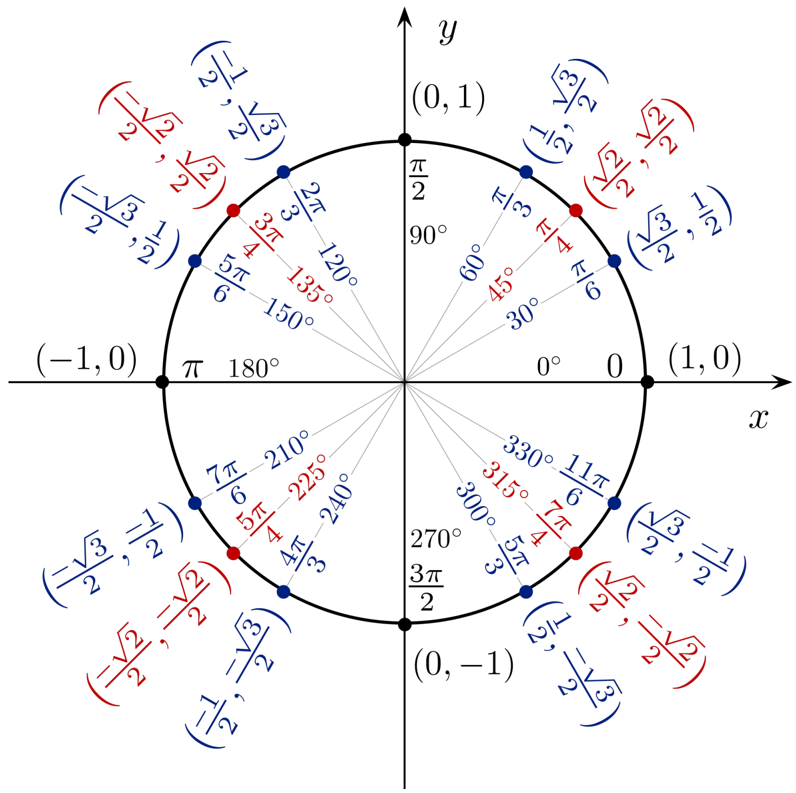




Some Examples

1. Simplify $\sin(\theta)\csc(\theta)\tan(\theta)$
2. Simplify $(\sec(\theta) + \tan(\theta))(\sec(\theta) - \tan(\theta))$
3. If θ is acute and $\tan(\theta) = \frac{3}{4}$ find:
   (a) $\cos(-\theta)$
   (b) $\sin(4\pi + \theta)$
4. Find $\tan(\pi - \theta)$ if $\tan(\theta) = 3$
5. Simplify $\frac{\cot^2(\theta)(\sec^2(\theta)-1)}{\sec^2(\theta)-\tan^2(\theta)+1}$

Answers:
1)$\tan(\theta)$ 2) 1 3) $\frac{4}{5}$ 4) -3
5) $\frac{1}{2}$



Unit Circle

A common tool used to evaluate trigonometric functions is called the *unit circle* (pictured above). Unit indicates that the radius is 1. In the picture, the x-coordinate is the value for $\cos(\theta)$ and the y-coordinate is the value for $\sin(\theta)$. To find $\tan(\theta)$ evaluate $\frac{y}{x}$. Starting in the first quadrant and moving counter-clockwise, you can use the acronym ASTC (All Students Take Calculus) to see which values are positive in what quadrant.
From the unit circle, you can also see an important conversion factor: $\pi\ radians = 180°$

More Examples

6. Convert 30° to radian measure.
7. Convert $\frac{3\pi}{2}$ to degrees
8. Find all solutions to $2\sin^2(\theta) - 3\sin(\theta) + 1 = 0$

Answers: 6) $\frac{\pi}{6}$ 7) 270° 8) $\frac{\pi}{6} + 2\pi n,\ \frac{5\pi}{6} + 2\pi n,\ \frac{\pi}{2} + 2\pi n$