Math 203: Final Exam Practice Problems

1 Test Format.

The exam will be 120 minutes. You can have two sides of an 8.5x11in paper, same as last exam. You will be asked to show all your work. If you do not show any work, then you will not receive any points. You will be awarded partial credit for partially correct answers.

These problems are only reviewing the last portion of lectures covered. Please review all previous materials for practice on the other topics. I'll post solutions to these practice problems before the test. You should attempt to solve all of these problems, lookup solutions to problems you do not understand, and ask questions if you need to.

2 Problems

The exam will consist of problems asking you about topics covered on previous exams as well as:

1. Functions of multiple variables
2. Partial derivatives
3. Finding critical points of functions of two variables
4. Finding the maximum and minimum of a function of two variables (second-derivative test)

3 Practice Problems.

These problems are intended to help you prepare for the test. Test problems on the above topics will be similar to, but not the same as, the problems below.

1. Let $h(x, y, z) = x^2y + x^2z^3 - \sqrt{x^3y^5z}$. Find $\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}$, and $\frac{\partial h}{\partial z}$.
2. Let $h(x, y, z) = \frac{x^2y^3}{z} + 2x^2$. Find $\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}$, and $\frac{\partial h}{\partial z}$.
3. Suppose $f(x, y) = 2x^2 - x^4 - y^2$. Find all the critical points for this function, and determine if they are maxima, minima, or saddle points. Show how you got your answer.
4. Let $g(x, y) = (x^2 - 1)(y^2 - 1)$.
   (a) Find all the critical points for $g(x, y)$.
   (b) Use the second derivative test to determine if each critical point from part (a) is a maximum, minimum, or saddle point. If it is a maximum or minimum, find the maximum/minimum value of g at that point. Must show work to receive credit.
5. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for each function $f(x, y)$.
   (a) $f(x, y) = 3x^2 + 2y^2 - 6xy$
   (b) $f(x) = x^2y + xy^2 - \frac{1}{x}$
   (c) $h(x) = e^{xy}$
   (d) $g(x) = \ln(x^2 + y^2)$

6. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$ for each function $f(x, y, z)$.

   (a) $f(x, y, z) = x + y + z$

   (b) $g(x, y, x) = x^2y^3z^4$

   (c) $h(x, y, z) = x^2z + \sqrt{xy}$

   (d) $j(x, y, z) = e^{x^2+y^2+z^2}$

7. Find the critical numbers for the given function of two variables and determine if they are maxima, minima, neither, or undetermined.

   (a) $f(x, y) = x^2 + y^2$

   (b) $f(x, y) = x^2 + y^2 - 2x - 4y + 2$

   (c) $f(x, y) = e^{-x^2-y^2}$