Math 203: Exam #1 Practice

1 Test Format.

The exam will be 50 minutes. If you do not show any work, then you will not receive any points. How much work should you show? *You are trying to convince me to give you points on a problem. Show me what you know.* You will be awarded partial credit for partially correct answers. The amount of partial credit depends on how many and how serious any errors are.

2 Topics

The list of topics this exam will cover will include (but not necessarily limited to):

1. Chapter 1: The Derivative
   (a) Finding equations for lines (especially tangent lines)
   (b) Definition of the derivative as slope of tangent line.
   (c) Calculating the derivative using the difference quotient.
   (d) Calculating the derivative using the derivative rules.
   (e) Calculating limits of functions using limit laws and determining if a limit exists.
   (f) Continuity of functions.
   (g) The derivative as a rate of change of a function.

I suggest reviewing homework problems, classwork problems, and lecture examples as well. I'll post solutions to these practice problems before the test. You should attempt to solve all of these problems, lookup solutions to problems you do not understand, and ask questions if you need to.

3 Practice Problems.

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.*

1. What is the derivative of a function? What does it tell you about $f(x)$? Try to give as many different answers to this question as you can.

2. Find the slope-intercept equation for the line...
   (a) that passes through the two points $(-2, 2)$ and $(3, 5)$.
   (b) with slope -1.5 and passes through $(1, -3.5)$.
   (c) parallel to the line $y = 3.7x + 1.4$ with y-intercept at $y = 2.8$.
   (d) that crosses the x-axis at the point $x = 10.5$ and crosses the y-axis at the point $y = 1$.

3. Use the limit definition of a derivative to find the derivative of each of the following functions:

(a) $g(t) = 5t^2$

(b) $h(s) = s^2 - 3s + 3$

(c) $f(x) = \frac{5}{x^2}$

(d) $w(z) = 2.1z + 6$

(e) $\ell(z) = \frac{1}{1-z}$

(f) $f(x) = \sqrt{x}$

4. Let $f(x) = \frac{1}{\sqrt[5]{x}}$. Find the equation for the tangent line to $f(x)$ at the point $(-1, -1)$.

5. Consider the function $f(x) = 5 - x^2$.

   (a) Find the equation for the secant line to the graph of $f(x)$ that passes through the points $(1, 4)$ and $(2, 1)$.

   (b) Find the equation for the tangent line to the graph of $f(x)$ at the point $(1, 4)$.

   (c) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2, 1)$.

6. Find the indicated limit. If the limit does not exist, write DNE, ∞, or $-\infty$ as appropriate.

(a) $\lim_{x \to 3} 4x^3$

(b) $\lim_{x \to 0} \frac{1}{x^3 - 1} + 1$

(c) $\lim_{x \to 1} \frac{3x - 4}{x^2 + x + 1}$

(d) $\lim_{x \to -1} \frac{x^2 + x - 2}{x^3 + 1}$

(e) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

(f) $\lim_{x \to -3} \sqrt{x - 5}$

(g) $\lim_{x \to \infty} \frac{2x^3 - 4x^2 + 5x}{17x^2 + 1}$

(h) $\lim_{x \to -\infty} \frac{3x^2 + 4}{x + 7}$

7. A coffee bean roaster gives a bulk discount for large purchases. For purchases up to 10 pounds, the company charges \$7.50 per pound. For purchases greater than 10 pounds, the company charges \$6.50 per pound. A simple representation of this revenue is the function

$$R(x) = \begin{cases} 7.50x & 0 \le x \le 10 \\ 6.50x & x > 10 \end{cases}$$

   (a) For what values of x is the revenue function above *not continuous*? How do you know?

   (b) Find a number C so that the function $R(x) = \begin{cases} 7.50x & 0 \le x \le 10 \\ 6.50x + C & x > 10 \end{cases}$ is continuous for all x.

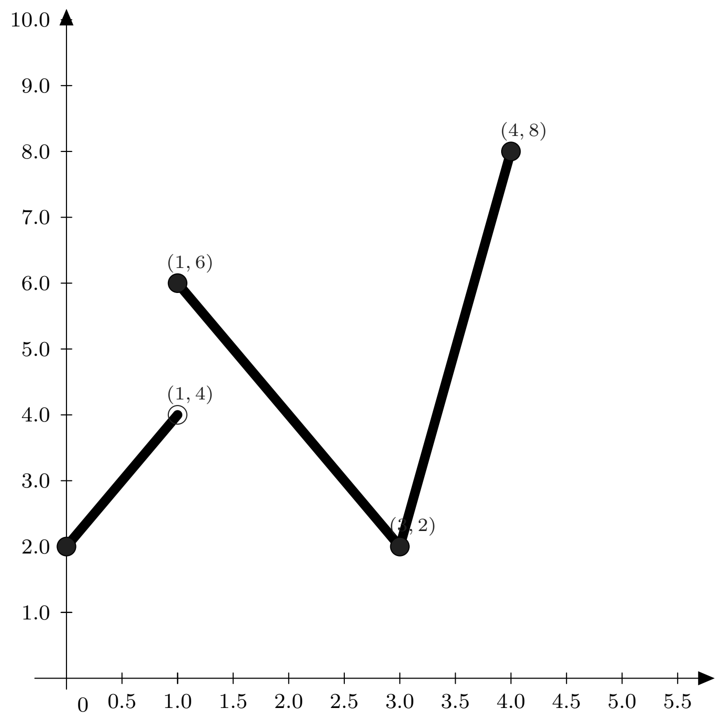
8. Suppose $\lim_{x \to 1} f(x) = \frac{2}{3}$ and $\lim_{x \to 1} g(x) = \frac{-5}{11}$/ Find $\lim_{x \to 1} \frac{f(x)}{g(x)}$.

9. Below is a graph of a function $f(x)$.

   (a) Find the domain and range of $f(x)$.

   (b) List the interval(s) on which f is continuous. At each point of discontinuity, find the limit from the left and the limit from the right.

   (c) Find a piecewise function to describe $f(x)$: $f(x) = \begin{cases} a(x) & \text{if } x < ... \\ b(x) & \text{if } ... \leq x < ... \\ \vdots & \\ etc. & \end{cases}$



10. Find the derivative. Show your solution step-by-step. You don't need to simplify your answer.

   (a) $f(x) = 7x^4 - \frac{4}{3}x^3 + \frac{1}{10}x^2 - \frac{8}{9}x + 40$

   (b) $f(x) = \sqrt[4]{x^3}$

   (c) $g(x) = \frac{6}{x^5} - \frac{8}{x}$

Word Problems.

11. A company does extensive market research to determine the optimal price for their product. They find out that:

   - If the price is $P = \$50$, the demand is $x = 600$ units in a week.
   - If the price is $P = \$40$, the demand is $x = 800$ units in a week.

   Find a linear function $P(x)$ that expresses the price P in terms of the demand x.

12. A disease is modeled using the function $p(t) = \frac{1}{3}(t^2 + t)$ where t is the number of days since the first case is discovered, and $p(t)$ is the percentage of the population that is infected.

    (a) What percentage of the population are infected after 5 days?

    (b) At what rate is the population being infected ager 5 days?

13. A company's total sales (in millions of dollars) t months from now are $S(t) = 2\sqrt{t} + 5$.

    (a) Find $S'(t)$.

    (b) Find $S(25)$ and $S'(25)$. Interpret the meaning of these two numbers.

14. A company sells widgets for \$5.00 each. For orders of 500 widgets or more, the company reduces the price to \$3.50 each. Write a piecewise function for the price of buying x widgets (piecewise means your function is of the form $f(x) = \begin{cases} a(x) & \text{if } x < A \\ b(x) & \text{if } x \geq B \end{cases}$