MATH 203: EXAM #2 PRACTICE

1 Test Format.

The exam will be 50 minutes. I guess you can have a 3×5 notecard, same as last exam. You will be asked to show all your work. If you do not show any work, then you will not receive any points. How much work should you show? *You are trying to convince me to give you points on a problem. Show me what you know.* You will be awarded partial credit for partially correct answers. The amount of partial credit depends on how many and how serious any errors are. You should show your work so I can determine how much partial credit to give you on a problem.

2 Problems

The exam will consist of problems asking you to do the following:

1. Find the derivative of several different functions, including using the product, quotient, chain, and implicit differentiation rules.
2. Use the graph of the derivative $f'(x)$ to answer questions about an unknown function $f(x)$.
3. Find and classify critical numbers for a function using the first derivative test and/or the second derivative test.
4. Sketch the graph of a function $f(x)$ using the techniques of curve sketching.
5. Optimizing revenue/cost/profit given the price/demand function $p(x)$.
6. Optimizing inventory control given Ordering Costs and Carrying Costs.
7. Other kinds of optimization problems.
8. Problems involving related rates

3 Practice Problems.

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem. You should take responsibility for your prep and use this review sheet to guide to help you study on your own.*

1. Give a definition for a *critical number*.
2. Suppose $x = a$ is a critical number for f. Explain how to use the first derivative $f'(x)$ to describe what happens at $f(a)$.
3. Suppose $x = a$ is a critical number for f. Explain how to use the second derivative $f''(x)$ to describe what happens at $f(a)$.
4. Explain the steps in solving an optimization word problem (there should be 4-5 steps involved, but the number of steps isn't important, so much as knowing the process.)
5. Find the derivative. Show your solution step-by-step. You don't need to simplify your answer.

(a) $f(x) = (x^2 - 1)(x - 1)$

(b) $f(x) = \frac{x^3+1}{1+x^2}$

(c) $h(x) = \frac{(x^2+1)^3+1}{1+(x^2+1)^2}$ (hint: look at your answer to (b))

(d) $g(x) = x^2\sqrt{x-1}$

6. For each function:

   (i) Find all critical numbers.

   (ii) Determine whether each critical number is a relative maximum, relative minimum, or neither.

   (iii) Find all inflection points for $f(x)$.

   (iv) Use other information about $f(x)$ to sketch a graph of $f(x)$. Label the (x, y)-coordinates of each extremum, intercept, and inflection point for full credit.

   (a) $f(x) = x^2 - 6x + 9$

   (b) $f(x) = 2x^2 - 7x + 3$

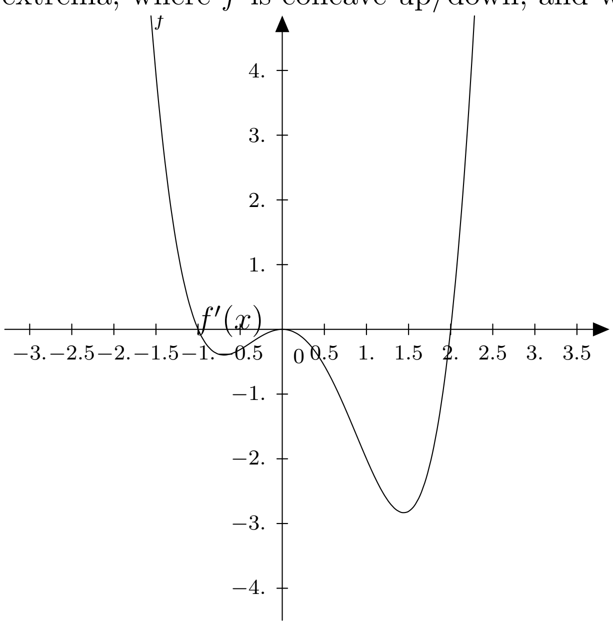
   (c) $f(x) = x^3 - x^2$

   (d) $f(x) = 3x^4 - 12x^3$

   (e) $f(x) = \sqrt[3]{x^2 - 3}$

   (f) $f(x) = \frac{x}{x^2+1}$

7. Below is a graph of the *derivative* of a function f. Use the graph to determine where f has relative extrema, where f is concave up/down, and where f has inflection points.



8. Find dy/dx for $(x^2 + y^2)^3 = 8x^2y^2$ at the point $(-1, 1)$. Use this to find the equation of the tangent line to the curve at $(-1, 1)$.

9. Find equations for the tangent line to $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-1, 0)$.

Word Problems.

10. Find two numbers whose sum is 32 and whose product is maximized.

11. Find two positive numbers whole product is 100, and whose sum is minimized.

12. A fence is to be built around a rectangular garden. The garden requires 100 square yards of area. Find the dimensions of the garden (length and width) that maximize the area.

13. A fence is to be built around a (different) rectangular garden. One side of the fence is to be built with brick, costing \$35.00 per yard. The other three sides are to be built using wooden planks, costing \$25.00 per yard. The yard still requires 100 square yards of area. Find the dimensions that minimize the cost of constructing the fence.

14. The price function for a particular commodity with demand level x is $p(x) = 9 - 0.03x$. Find the production level that maximizes the revenue.

15. A company that produces electronic components expects to sell 4000 units this year. They can produce the units in production runs, but each production run costs \$250 to set up. After each production, it has to pay \$2.00 per unit to store the inventory, until they sell out (inventory cost is based on the *average* inventory between production runs). How many production runs should the company do in order to minimize the Inventory costs?

16. Air is being pumped into a spherical balloon at a rate of 7 cm^3 per second. How fast is the *radius* of the balloon increasing at the instant the *volume* equals 36π? (The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$; you may write your answer in terms of π if you like).

17. The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

18. A ice cube is melting at a rate of 4cm^3 per minute. How fast is the length of the side decreasing when the side is 2cm? Recall the volume of a cube with side x is $V = x^3$.

19. Suppose the price p and demand x of a commodity are related by the equation $100x^2 + 9p^2 = 3600$. Due to a surplus, the price p of the commodity is dropping at a rate of \$0.15 per week, causing the demand to increase. How fast is the demand increasing when the price p is equal to \$15?