Math 203 Fall 2018 Professor MG
Classwork 4 Name: ________________________________

Show all work and circle/box your final answer. All answers must be simplified unless stated otherwise.

SOME REMINDERS:

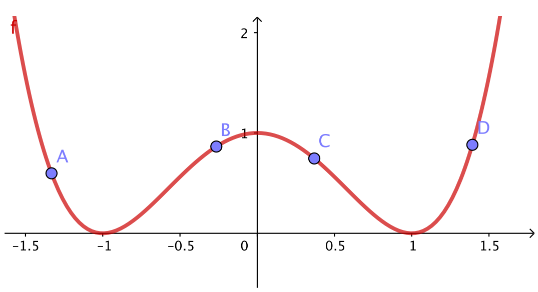
The first and second derivatives tell you about the graph of a function. The first derivative tells you where the graph is increasing or decreasing, and where to look for relative extrema. The second derivative tells you whether the graph is concave up or concave down. The critical numbers where $f' = 0$ or f' is undefined

If the sign of	$f'(x)$	$f''(x)$
is > 0 then	f is increasing	f is concave up
< 0	f is decreasing	f is concave down
$= 0$	critical number	important number
is undefined	critical number	important number

With some care we can use this information to *draw the graph* of $f(x)$. This requires understanding how f, f' and f'' interact.

CLASSWORK PROBLEMS

1. (*0 points*) Match each point on the left to the description on the right.



(a) $f' > 0$ and $f'' < 0$

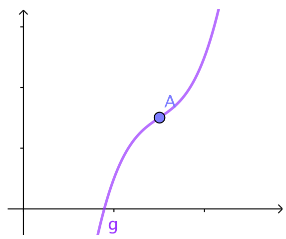
(b) $f' < 0$ and $f'' < 0$

(c) $f' > 0$ and $f'' > 0$

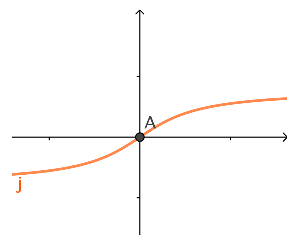
(d) $f' < 0$ and $f'' > 0$

2. (*0 points*) Look at the inflection points below and determine the sign of f'' on either side of the inflection point, and also the sign of f' at the inflection point.

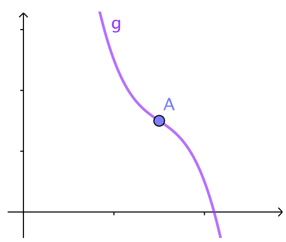
(a)



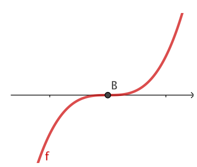
(c)



(b)



(d)



3. (*0 points*) Find intervals of increase/decrease and all relative extrema of the following:

$$f(x) = x^3 + 6x^2 + 9x - 8$$

4. (*0 points*) Find intervals of concavity and any inflection points of $f(x) = x^4 - 4x^3$

5. (*0 points*) Graph $f(x) = 2x^3 - 3x^2 + x$ using the curve sketching steps listed in class. There are worked through examples of curve sketching on the next page for your reference.

CURVE SKETCHING EXAMPLES

1. Graph $f(x) = 3x^2 - 6x$

Solution

Domain: There are no restrictions so the domain is $(-\infty, \infty)$

x-intercepts:

$$f(x) = 0 \Leftrightarrow 3x^2 - 6x = 0$$
$$\Leftrightarrow 3x(x-2) = 0$$
$$\Leftrightarrow x = 0, 2$$

y-intercept:

$$f(0) = 0$$

Vertical Asymptote: The function is never undefined so there is no vertical asymptote.

Horizontal Asymptote:

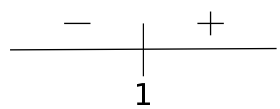
$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} 3x^2 = \infty$$

So there is no horizontal asymptote.

First Derivative Test:

$$f'(x) = 6x - 6$$
$$= 6(x-1)$$

Thus our key point is $x = 1$. Testing the intervals we have the following number line for $f'(x)$:



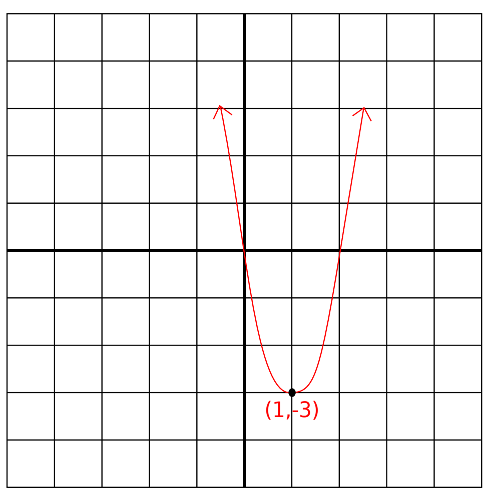
Thus we have that f is increasing on $(1, \infty)$, decreasing on $(-\infty, 1)$, and there is a relative minimum at $x = 1$.

$$f(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$$

So our relative minimum is $(1, -3)$

Second Derivative Test:

$$f''(x) = 6 \text{ so } f \text{ is always concave up}$$



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2. Sketch the graph of $\dfrac{x-1}{x+2}$

Solution

Domain: x cannot be -2 so the domain is $(-\infty,-2)\cup(-2,\infty)$

x-intercept:

$$\begin{aligned}\frac{x-1}{x+2}=0 &\Leftrightarrow x-1=0\\ &\Leftrightarrow x=1\end{aligned}$$

y-intercept:

Plugging in $x=0$ we have

$$\frac{0-1}{0+2}=-\frac{1}{2}$$

Horizontal Asymptote:

$$\begin{aligned}\lim_{x\to\pm\infty}\frac{x-1}{x+2} &= \lim_{x\to\pm\infty}\frac{x/x-1/x}{x/x+2/x}\\ &= \lim_{x\to\pm\infty}\frac{1-1/x}{1+2/x}\\ &= \frac{1-0}{1+0}\\ &= 1\end{aligned}$$

So there is a horizontal asymptote of $y=1$

Vertical Asymptote:

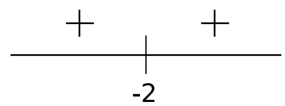
$$\frac{x-1}{x+2} \text{ is undefined} \Leftrightarrow x+2=0 \Leftrightarrow x=-2$$

Thus the vertical asymptote is $x=-2$

First Derivative Test:

$$\begin{aligned}\frac{d}{dx}\left(\frac{x-1}{x+2}\right) &= \frac{(x+2)\frac{d}{dx}(x-1)-(x-1)\frac{d}{dx}(x+2)}{(x+2)^2}\\ &= \frac{\cancel{x}+2-\cancel{x}+1}{(x+2)^2}\\ &= \frac{3}{(x+2)^2}\end{aligned}$$

So our key point is $x = -2$. Testing the intervals we have

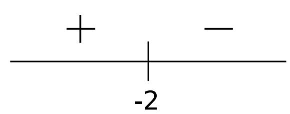


Thus the function is always increasing and there is no relative extrema.

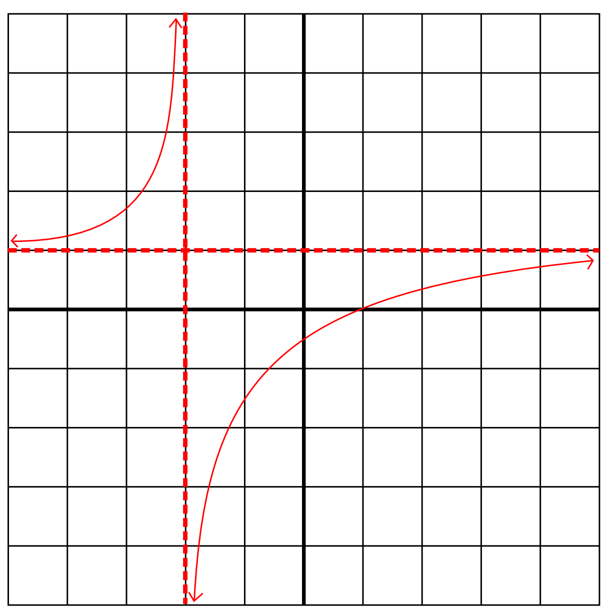
<u>Second Derivative Test:</u>

$$\begin{aligned}\frac{d}{dx}\left(\frac{3}{(x+2)^2}\right) &= \frac{d}{dx}(3(x+2)^{-2}) \\ &= 3(-2)(x+2)^{-3} \\ &= -\frac{6}{(x+2)^2}\end{aligned}$$

Thus the important point is $x = -2$. Checking the intervals we have



So the function is concave up on $(-\infty, -2)$ and concave down on $(-2, \infty)$. However, there is no inflection point since the function is undefined at $x = -2$



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