Check Your Understanding 0.1

1. Is the point $(3, 12)$ on the graph of the function $g(x) = x^2 + 5x - 10$?

2. Sketch the graph of the function $h(t) = t^2 - 2$.

EXERCISES 0.1

Draw the following intervals on the number line.

1. $[-1, 4]^*$
2. $(4, 3\pi)^*$
3. $[-2, \sqrt{2})^*$
4. $[1, \frac{3}{2}]^*$
5. $(-\infty, 3)^*$
6. $(4, \infty)^*$

Use intervals to describe the real numbers satisfying the inequalities in Exercises 7–12.

7. $2 \le x < 3$ $[2, 3)$
8. $-1 < x < \frac{3}{2}$ $(-1, 3/2)$
9. $x < 0$ and $x \ge -1$ $[-1, 0)$
10. $x \ge -1$ and $x < 8$ $[-1, 8)$
11. $x < 3$ $(-\infty, 3)$
12. $x \ge \sqrt{2}$ $[\sqrt{2}, \infty)$

13. If $f(x) = x^2 - 3x$, find $f(0)$, $f(5)$, $f(3)$, and $f(-7)$. 0, 10, 0, 70

14. If $f(x) = x^3 + x^2 - x - 1$, find $f(1)$, $f(-1)$, $f(\frac{1}{2})$, and $f(a)$.

15. If $g(t) = t^3 - 3t^2 + t$, find $g(2)$, $g(-\frac{1}{2})$, $g(\frac{2}{3})$, and $g(a)$.

16. If $h(s) = s/(1+s)$, find $h(\frac{1}{2})$, $h(-\frac{3}{2})$, and $h(a+1)$. $1/3$, 3, $(a+1)/(a+2)$

17. If $f(x) = x^2 - 2x$, find $f(a+1)$ and $f(a+2)$. $a^2 - 1$, $a^2 + 2a$

18. If $f(x) = x^2 + 4x + 3$, find $f(a-1)$ and $f(a-2)$.

19. **Computer Sales** An office supply firm finds that the number of laptop computers sold in year x is given approximately by the function $f(x) = 150 + 2x + x^2$, where $x = 0$ corresponds to 2010.
   (a) What does $f(0)$ represent? Sales in 2010
   (b) Find the number of laptops sold in 2016. $f(6) = 198$

20. **Response of a Muscle** When a solution of acetylcholine is introduced into the heart muscle of a frog, it diminishes the force with which the muscle contracts. The data from experiments of the biologist A. J. Clark are closely approximated by a function of the form

$$R(x) = \frac{100x}{b + x}, \qquad x \ge 0,$$

where x is the concentration of acetylcholine (in appropriate units), b is a positive constant that depends on the particular frog, and $R(x)$ is the response of the muscle to the acetylcholine, expressed as a percentage of the maximum possible effect of the drug.
   (a) Suppose that $b = 20$. Find the response of the muscle when $x = 60$. $R(60) = 75$
   (b) Determine the value of b if $R(50) = 60$, that is, if a concentration of $x = 50$ units produces a 60% response. $b = 100/3$

In Exercises 21–28, describe the domain of the function.

21. $f(x) = \dfrac{8x}{(x-1)(x-2)}$ $x \ne 1, 2$
22. $f(t) = \dfrac{1}{\sqrt{t}}$ $t > 0$ $(0, \infty)$
23. $g(x) = \dfrac{1}{\sqrt{3-x}}$ $x < 3$
24. $g(x) = \dfrac{4}{x(x+2)}$
25. $f(x) = \dfrac{3x-5}{x^2+x-6}$ $x \ne -3, 2$
26. $f(x) = \dfrac{1}{3x^2+1}$ All x
27. $f(x) = \sqrt{2x+7} + \sqrt{x}$ $x \ge 0$
28. $f(x) = \dfrac{\sqrt{2x+1}}{\sqrt{1-x}}^*$

$*$ indicates answers that are in the back of the book. 14. $0, 0, -9/8, a^3 + a^2 - a - 1$ 15. $g(2) = -2$, $g(-1/2) = -11/8$, $g(2/3) = -10/27 = -.37037$, $g(a) = a^3 - 3a^2 + a$ 18. $f(a-1) = a^2 + 2a$, $f(a-2) = a^2 - 1$ 24. $x \ne 0$, $x \ne -2$ $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

Check Your Understanding 0.3

1. Let $f(x) = x^5$, $g(x) = x^3 - 4x^2 + x - 8$.
   (a) Find $f(g(x))$. (b) Find $g(f(x))$.

2. Let $f(x) = x^2$. Calculate $\dfrac{f(1+h) - f(1)}{h}$ and simplify.

EXERCISES 0.3

Let $f(x) = x^2 + 1$, $g(x) = 9x$, and $h(x) = 5 - 2x^2$.
Calculate the following functions.

1. $f(x) + g(x)$ $x^2 + 9x + 1$
2. $f(x) - h(x)$ $3x^2 - 4$
3. $f(x)g(x)$ $9x^3 + 9x$
4. $g(x)h(x)$ $45x - 18x^3$
5. $\dfrac{f(t)}{g(t)}$ $\dfrac{t^2+1}{9t}$
6. $\dfrac{g(t)}{h(t)}$ $\dfrac{9t}{5-2t^2}$

In Exercises 7–12, express $f(x) + g(x)$ as a rational function. Carry out all multiplications.

7. $f(x) = \dfrac{2}{x-3}$, $g(x) = \dfrac{1}{x+2}$ $\dfrac{3x+1}{x^2-x-6}$
8. $f(x) = \dfrac{3}{x-6}$, $g(x) = \dfrac{-2}{x-2}$ $\dfrac{x+6}{x^2-8x+12}$
9. $f(x) = \dfrac{x}{x-8}$, $g(x) = \dfrac{-x}{x-4}$ $\dfrac{4x}{x^2-12x+32}$
10. $f(x) = \dfrac{-x}{x+3}$, $g(x) = \dfrac{x}{x+5}$ $\dfrac{-2x}{x^2+8x+15}$
11. $f(x) = \dfrac{x+5}{x-10}$, $g(x) = \dfrac{x}{x+10}$ $\dfrac{2x^2+5x+50}{x^2-100}$
12. $f(x) = \dfrac{x+6}{x-6}$, $g(x) = \dfrac{x-6}{x+6}$ $\dfrac{2x^2+72}{x^2-36}$

Let $f(x) = \dfrac{x}{x-2}$, $g(x) = \dfrac{5-x}{5+x}$, and $h(x) = \dfrac{x+1}{3x-1}$.
Express the following as rational functions.

13. $f(x) - g(x)$
14. $f(t) - h(t)$ $\dfrac{2t^2+2}{3t^2-7t+2}$
15. $f(x)g(x)$ $\dfrac{-x^2+5x}{x^2+3x-10}$
16. $g(x)h(x)$ $\dfrac{-x^2+4x+5}{3x^2+14x-5}$
17. $\dfrac{f(x)}{g(x)}$ $\dfrac{x^2+5x}{-x^2+7x-10}$
18. $\dfrac{h(s)}{f(s)}$ $\dfrac{s^2-s-2}{3s^2-s}$
19. $f(x+1)g(x+1)$
20. $f(x+2) + g(x+2)$ $\dfrac{12x+14}{x^2+7x}$
21. $\dfrac{g(x+5)}{f(x+5)}$ $\dfrac{-x^2-3x}{x^2+15x+50}$
22. $f\left(\dfrac{1}{t}\right)$ $\dfrac{1}{-2t+1}$
23. $g\left(\dfrac{1}{u}\right)$ $\dfrac{5u-1}{5u+1}, u \neq 0$
24. $h\left(\dfrac{1}{x^2}\right)$ $\dfrac{1+x^2}{3-x^2}$

Let $f(x) = x^6$, $g(x) = \dfrac{x}{1-x}$, and $h(x) = x^3 - 5x^2 + 1$.
Calculate the following functions.

25. $f(g(x))$
26. $h(f(t))$ $t^{18} - 5t^{12} + 1$
27. $h(g(x))$
28. $g(f(x))$ $\dfrac{x^6}{1-x^6}$
29. $g(h(t))$ $\dfrac{t^3-5t^2+1}{-t^3+5t^2}$
30. $f(h(x))$ $(x^3-5x^2+1)^6$
31. If $f(x) = x^2$, find $f(x+h) - f(x)$ and simplify. $2xh + h^2$
32. If $f(x) = 1/x$, find $f(x+h) - f(x)$ and simplify. $\dfrac{-h}{x(x+h)}$
33. If $g(t) = 4t - t^2$, find $\dfrac{g(t+h) - g(t)}{h}$ and simplify. $4 - 2t - h$
34. If $g(t) = t^3 + 5$, find $\dfrac{g(t+h) - g(t)}{h}$ and simplify. $3t^2 + 3th + h^2$

35. **Cost** After t hours of operation, an assembly line has assembled $A(t) = 20t - \frac{1}{2}t^2$ power lawn mowers, $0 \le t \le 10$. Suppose that the factory's cost of manufacturing x units is $C(x)$ dollars, where $C(x) = 3000 + 80x$.
   (a) Express the factory's cost as a (composite)function of the number of hours of operation of the assembly line. $C(f(t)) = 3000 + 1600t - 40t^2$
   (b) What is the cost of the first 2 hours of operation? $6040

36. **Cost** During the first $\frac{1}{2}$ hour, the employees of a machine shop prepare the work area for the day's work. After that, they turn out 10 precision machine parts per hour, so the output after t hours is $f(t)$ machine parts, where $f(t) = 10\left(t - \frac{1}{2}\right) = 10t - 5$, $\frac{1}{2} \le t \le 8$. The total cost of producing x machine parts is $C(x)$ dollars, where $C(x) = .1x^2 + 25x + 200$.
   (a) Express the total cost as a (composite) function of t.*
   (b) What is the cost of the first 4 hours of operation?*

37. **Conversion Scales** Table 1 shows a conversion table for men's hat sizes for three countries. The function $g(x) = 8x + 1$ converts from British sizes to French sizes, and the function $f(x) = \frac{1}{8}x$ converts from French sizes to U.S. sizes. Determine the function $h(x) = f(g(x))$ and give its interpretation.*

TABLE 1 Conversion Table for Men's Hat Sizes

Britain	$6\frac{1}{2}$	$6\frac{5}{8}$	$6\frac{3}{4}$	$6\frac{7}{8}$	7	$7\frac{1}{8}$	$7\frac{1}{4}$	$7\frac{3}{8}$
France	53	54	55	56	57	58	59	60
U.S.	$6\frac{5}{8}$	$6\frac{3}{4}$	$6\frac{7}{8}$	7	$7\frac{1}{8}$	$7\frac{1}{4}$	$7\frac{3}{8}$	$7\frac{1}{2}$

Technology Exercises

38. Let $f(x) = x^2$. Graph the functions $f(x+1)$, $f(x-1)$, $f(x+2)$, and $f(x-2)$. Make a guess about the relationship between the graph of a general function $f(x)$ and the graph of $f(g(x))$, where $g(x) = x + a$ for some constant a. Test your guess on the functions $f(x) = x^3$ and $f(x) = \sqrt{x}$.*

39. **Shifting a Graph** Let $f(x) = x^2$. Graph the functions $f(x) + 1$, $f(x) - 1$, $f(x) + 2$, and $f(x) - 2$. Make a guess about the relationship between the graph of a general function $f(x)$ and the graph of $f(x) + c$ for some constant c. Test your guess on the functions $f(x) = x^3$ and $f(x) = \sqrt{x}$.*

40. Based on the results of Exercises 38 and 39, sketch the graph of $f(x) = (x-1)^2 + 2$ without using a graphing calculator. Check your result with a graphing calculator.

41. Based on the results of Exercises 38 and 39, sketch the graph of $f(x) = (x+2)^2 - 1$ without using a graphing calculator. Check your result with a graphing calculator.

* indicates answers that are in the back of the book.

13. $\dfrac{2x^2-2x+10}{x^2+3x-10}$ 19. $\dfrac{-x^2+3x+4}{x^2+5x-6}$ 25. $\left(\dfrac{x}{1-x}\right)^6$ 27. $\left(\dfrac{x}{1-x}\right)^3 - 5\left(\dfrac{x}{1-x}\right)^2 + 1$

you view the polynomial on the domain $[-M, M]$, you can usually find a smaller domain that also contains all the zeros.

Check Your Understanding 0.4

1. Solve the equation $x - \dfrac{14}{x} = 5$.

2. Use the quadratic formula to solve $7x^2 - 35x + 35 = 0$.

EXERCISES 0.4

Use the quadratic formula to find the zeros of the functions in Exercises 1–6.

1. $f(x) = 2x^2 - 7x + 6$ $2, 3/2$

2. $f(x) = 3x^2 + 2x - 1$ $x = -1, 1/3$

3. $f(t) = 4t^2 - 12t + 9$ $3/2$

4. $f(x) = \frac{1}{4}x^2 + x + 1$ $x = -2$

5. $f(x) = -2x^2 + 3x - 4$ No zeros

6. $f(a) = 11a^2 - 7a + 1$ $(7 \pm \sqrt{5})/22$

Use the quadratic formula to solve the equations in Exercises 7–12.

7. $5x^2 - 4x - 1 = 0$ $1, -1/5$

8. $x^2 - 4x + 5 = 0$ No soln

9. $15x^2 - 135x + 300 = 0$ $5, 4$

10. $z^2 - \sqrt{2}z - \frac{5}{4} = 0$

11. $\frac{3}{2}x^2 - 6x + 5 = 0$ $2 + \sqrt{6}/3, 2 - \sqrt{6}/3$

12. $9x^2 - 12x + 4 = 0$ $x = 2/3$

Factor the polynomials in Exercises 13–30.

13. $x^2 + 8x + 15$ $(x + 5)(x + 3)$

14. $x^2 - 10x + 16$ $(x - 2)(x - 8)$

15. $x^2 - 16$ $(x - 4)(x + 4)$

16. $x^2 - 1$ $(x + 1)(x - 1)$

17. $3x^2 + 12x + 12$ $3(x + 2)^2$

18. $2x^2 - 12x + 18$ $2(x - 3)^2$

19. $30 - 4x - 2x^2$

20. $15 + 12x - 3x^2$ $-3(x - 5)(x$

21. $3x - x^2$ $x(3 - x)$

22. $4x^2 - 1$ $(2x + 1)(2x - 1)$

23. $6x - 2x^3$

24. $16x + 6x^2 - x^3$ $-x(x - 8)(x$

25. $x^3 - 1$ $(x - 1)(x^2 + x + 1)$

26. $x^3 + 125$ $(x + 5)(x^2 - 5x + 2$

27. $8x^3 + 27$

28. $x^3 - \frac{1}{8}$ $(x - 1/2)(x^2 + x/2 +$

29. $x^2 - 14x + 49$ $(x - 7)^2$

30. $x^2 + x + \frac{1}{4}$ $(x + 1/2)^2$

Find the points of intersection of the pairs of curves in Exercises 31–38.

31. $y = 2x^2 - 5x - 6$, $y = 3x + 4$ $(-1, 1), (5, 19)$

32. $y = x^2 - 10x + 9$, $y = x - 9$ $(9, 0), (2, -7)$

33. $y = x^2 - 4x + 4$, $y = 12 + 2x - x^2$ $(-1, 9), (4, 4)$

34. $y = 3x^2 + 9$, $y = 2x^2 - 5x + 3$ $(-3, 36), (-2, 21)$

35. $y = x^3 - 3x^2 + x$, $y = x^2 - 3x$ $(0, 0), (2, -2)$

10. $(\sqrt{2} \pm \sqrt{7})/2$ 19. $-2(x - 3)(x + 5)$ 23. $-2x(x - \sqrt{3})(x + \sqrt{3})$ 27. $(2x + 3)(4x^2 - 6x + 9)$