INCORPORATING TECHNOLOGY

Scientific Notation

By default, your TI-83/84-type calculator will display the result of a calculation in 10 digits. For example, if asked to calculate 1/3, your calculator will return .3333333333, and 7/3 returns 2.333333333. In each case, 10 digits are given in the answer.

However, if the answer cannot display 10 digits (or if the absolute value is less than .001), your calculator will express the answer in scientific notation. Scientific notation expresses numbers in two parts. The significant digits display with one digit to the left of the decimal. The appropriate power of 10 displays to the right of **E**, as in **2.5E-4**. This stands for 2.5×10^{-4} or 0.00025. Similarly, **1E12** stands for 1×10^{12} or 1,000,000,000,000. (*Note*: Multiplying a number by 10^{-4} moves the decimal point four places to the left, and multiplying by 10^{12} moves the decimal point 12 places to the right.)

Check Your Understanding 0.5

1. Compute the following.
   (a) -5^2 (b) $16^{.75}$

2. Simplify the following.
   (a) $(4x^3)^2$ (b) $\frac{\sqrt[3]{x}}{x^3}$ (c) $\frac{2 \cdot (x+5)^6}{x^2+10x+25}$

EXERCISES 0.5

In Exercises 1–28, compute the numbers.

1. 3^3 27
2. $(-2)^3$ -8
3. 1^{100} 1
4. 0^{25} 0
5. $(.1)^4$ .0001
6. $(100)^4$
7. -4^2 -16
8. $(.01)^3$ 0.000001
9. $(16)^{1/2}$ 4
10. $(27)^{1/3}$ 3
11. $(.000001)^{1/3}$ .01
12. $\left(\frac{1}{125}\right)^{1/3}$ 1/5
13. 6^{-1} 1/6
14. $\left(\frac{1}{2}\right)^{-1}$ 2
15. $(.01)^{-1}$ 100
16. $(-5)^{-1}$ $-1/5$
17. $8^{4/3}$ 16
18. $16^{3/4}$ 8
19. $(25)^{3/2}$ 125
20. $(27)^{2/3}$ 9
21. $(1.8)^0$ 1
22. $9^{1.5}$ 27
23. $16^{.5}$ 4
24. $(81)^{.75}$ 27
25. $4^{-1/2}$ 1/2
26. $\left(\frac{1}{8}\right)^{-2/3}$ 4
27. $(.01)^{-1.5}$ 1000
28. $1^{-1.2}$ 1

In Exercises 29–40, use the laws of exponents to compute the numbers.

29. $5^{1/3} \cdot 200^{1/3}$ 10
30. $(3^{1/3} \cdot 3^{1/6})^6$ 27
31. $6^{1/3} \cdot 6^{2/3}$ 6
32. $(9^{4/5})^{5/8}$ 3
33. $\frac{10^4}{5^4}$ 16
34. $\frac{3^{5/2}}{3^{1/2}}$ 9
35. $(2^{1/3} \cdot 3^{2/3})^3$ 18
36. $20^{.5} \cdot 5^{.5}$ 10
37. $\left(\frac{8}{27}\right)^{2/3}$ 4/9
38. $(125 \cdot 27)^{1/3}$ 15
39. $\frac{7^{4/3}}{7^{1/3}}$ 7
40. $(6^{1/2})^0$ 1

In Exercises 41–70, use the laws of exponents to simplify the algebraic expressions. Your answer should not involve parentheses or negative exponents.

41. $(xy)^6$ x^6y^6
42. $(x^{1/3})^6$ x^2
43. $\frac{x^4 \cdot y^5}{xy^2}$ x^3y^3
44. $\frac{1}{x^{-3}}$ x^3
45. $x^{-1/2}$ $\frac{1}{x^{1/2}}$
46. $(x^3 \cdot y^6)^{1/3}$ xy^2
47. $\left(\frac{x^4}{y^2}\right)^3$ x^{12}/y^6
48. $\left(\frac{x}{y}\right)^{-2}$ y^2/x^2
49. $(x^3y^5)^4$ $x^{12}y^{20}$
50. $\sqrt{1+x}\,(1+x)^{3/2}$
51. $x^5 \cdot \left(\frac{y^2}{x}\right)^3$ x^2y^6

6. 100,000,000 50. $1+2x+x^2$ 77. $1/x^{5/6}$ 78. $1/x^{5/9}$

52. $x^{-3} \cdot x^7$ x^4
53. $(2x)^4$ $16x^4$
54. $\frac{-3x}{15x^4}$ $-1/5x^3$
55. $\frac{-x^3y}{-xy}$ x^2
56. $\frac{x^3}{y^{-2}}$ x^3y^2
57. $\frac{x^{-4}}{x^3}$ $1/x^7$
58. $(-3x)^3$ $-27x^3$
59. $\sqrt[3]{x} \cdot \sqrt[3]{x^2}$ x
60. $(9x)^{-1/2}$ $1/3\sqrt{x}$
61. $\left(\frac{3x^2}{2y}\right)^3$ $27x^6/8y^3$
62. $\frac{x^2}{x^5y}$ $1/x^3y$
63. $\frac{2x}{\sqrt{x}}$ $2\sqrt{x}$
64. $\frac{1}{yx^{-5}}$ x^5/y
65. $(16x^8)^{-3/4}$ $1/8x^6$
66. $(-8y^9)^{2/3}$ $4y^6$
67. $\sqrt{x}\left(\frac{1}{4x}\right)^{5/2}$ $1/32x^2$
68. $\frac{(25xy)^{3/2}}{x^2y}$ $125\sqrt{y}/\sqrt{x}$
69. $\frac{(-27x^5)^{2/3}}{\sqrt[3]{x}}$ $9x^3$
70. $(-32y^{-5})^{3/5}$ $-8/y^3$

Let $f(x) = \sqrt[3]{x}$ and $g(x) = \frac{1}{x^2}$. Calculate the following functions. Take $x > 0$.

71. $f(x)g(x)$ $1/x^{5/3}$
72. $\frac{f(x)}{g(x)}$ $x^{7/3}$
73. $\frac{g(x)}{f(x)}$ $1/x^{7/3}$
74. $[f(x)]^3 g(x)$ $1/x$
75. $[f(x)g(x)]^3$ $1/x^5$
76. $\sqrt{\frac{f(x)}{g(x)}}$ $x^{7/6}$
77. $\sqrt{f(x)g(x)}$
78. $\sqrt[3]{f(x)g(x)}$
79. $f(g(x))$ $1/x^{2/3}$
80. $g(f(x))$ $1/x^{2/3}$
81. $f(f(x))$ $x^{1/9}$
82. $g(g(x))$ x^4

The expressions in Exercises 83–88 may be factored as shown. Find the missing factors.

83. $\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}(\quad)$ $x-1$
84. $2x^{2/3} - x^{-1/3} = x^{-1/3}(\quad)$ $2x-1$
85. $x^{-1/4} + 6x^{1/4} = x^{-1/4}(\quad)$ $1+6x^{1/2}$
86. $\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \sqrt{xy}(\quad)$ $1/y - 1/x$
87. Explain why $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$. $a^{1/2} \cdot b^{1/2} = (ab)^{1/2}$ (Law 5)
88. Explain why $\sqrt{a}/\sqrt{b} = \sqrt{a/b}$. Law 6

In Exercises 89–96, evaluate $f(4)$.

89. $f(x) = x^2$ 16

90. $f(x) = x^3$ 64

91. $f(x) = x^{-1}$ 1/4

92. $f(x) = x^{1/2}$ 2

93. $f(x) = x^{3/2}$ 8

94. $f(x) = x^{-1/2}$ 1/2

95. $f(x) = x^{-5/2}$ 1/32

96. $f(x) = x^0$ 1

Calculate the compound amount from the given data in Exercises 97–104.

97. principal = \$500, compounded annually, 6 years, annual rate = 6% \$709.26

98. principal = \$700, compounded annually, 8 years, annual rate = 8% \$1295.65

99. principal = \$50,000, compounded quarterly, 10 years, annual rate = 9.5% \$127,857.61

100. principal = \$20,000, compounded quarterly, 3 years, annual rate = 12% \$28,515.22

101. principal = \$100, compounded monthly, 10 years, annual rate = 5% \$164.70

102. principal = \$500, compounded monthly, 1 year, annual rate = 4.5% \$522.97

103. principal = \$1500, compounded daily, 1 year, annual rate = 6% \$1592.75

104. principal = \$1500, compounded daily, 3 years, annual rate = 6% \$1795.80

105. **Annual Compound** Assume that a couple invests \$100[0] upon the birth of their daughter. Assume that the investment earns 6.8% compounded annually. What will the investment be worth on the daughter's 18th birthday? \$3268.00

106. **Annual Compound with Deposits** Assume that a couple invests \$4000 each year for 4 years in an investment that earns 8% compounded annually. What will the value of the investment be 8 years after the first amount is invested? \$26,483.83

107. **Quarterly Compound** Assume that a \$500 investment earns interest compounded quarterly. Express the value of the investment after 1 year as a polynomial in the annual rate of interest r.

108. **Semiannual Compound** Assume that a \$1000 investment earns interest compounded semiannually. Express the value of the investment after 2 years as a polynomial in the annual rate of interest r.

109. **Velocity** When a car's brakes are slammed on at a speed of x miles per hour, the stopping distance is $\frac{1}{20}x^2$ feet. Show that when the speed is doubled the stopping distance increases fourfold.
$1/20(2x)^2 = 1/20(4x^2) = 4(1/2$…

Technology Exercises

In Exercises 110–113, convert the numbers from graphing calculator form to standard form (that is, without E).

110. 5E-5 0.00005

111. 8.103E-4 .0008103

112. 1.35E13 13,500,000,000,000

113. 8.23E-6 .00000823

107. $500/256(256 + 256r + 96r^2 + 16r^3 + r^4) = A$ 108. $125/2(16 + 32r + 24r^2 + 8r^3 + r^4) = A$

Solutions to Check Your Understanding 0.5

1. (a) $-5^2 = -25$. [Note that -5^2 is the same as $-(5^2)$. This number is different from $(-5)^2$, which equals 25. Whenever there are no parentheses, apply the exponent first and then apply the other operations.]

(b) Since $.75 = \frac{3}{4}$, $16^{.75} = 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$.

2. (a) Apply Law 5 with $a = 4$ and $b = x^3$. Then, use Law 4.

$$(4x^3)^2 = 4^2 \cdot (x^3)^2 = 16 \cdot x^6$$

[A common error is to forget to square the 4. If that had been our intent, we would have asked for $4(x^3)^2$.]

(b) $$\frac{\sqrt[3]{x}}{x^3} = \frac{x^{1/3}}{x^3} = x^{(1/3)-3} = x^{-8/3}$$

[The answer can also be given as $1/x^{8/3}$.] When simplifying expressions involving radicals, it is usually a good idea to first convert the radicals to exponents.

(c) $$\frac{2(x+5)^6}{x^2+10x+25} = \frac{2\cdot(x+5)^6}{(x+5)^2} = 2(x+5)^{6-2} = 2(x+5)^4$$

[Here, the third law of exponents was applied to $(x+5)$. The laws of exponents apply to any algebraic expression.]

0.6 Functions and Graphs in Applications

The key step in solving many applied problems in this text is to construct appropriate functions or equations. Once this is done, the remaining mathematical steps are usually straightforward. This section focuses on representative applied problems and reviews skills needed to set up and analyze functions, equations, and their graphs.

Geometric Problems Many examples and exercises in the text involve dimensions, areas, or volumes of objects similar to those in Fig. 1. When a problem involves a plane figure, such as a rectangle or circle, we must distinguish between the perimeter and the *area* of the figure. The perimeter of a figure, or "distance around" the fi…

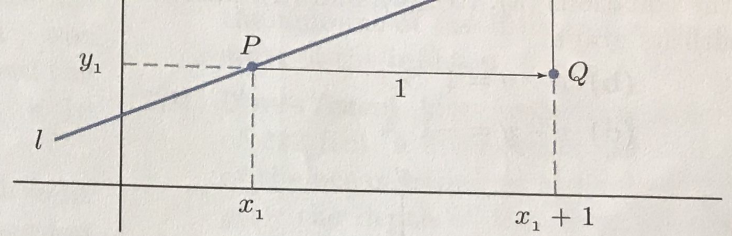


Figure 8

INCORPORATING *TECHNOLOGY* At the end of each section in Chapter 0, we reviewed in detail many very useful techniques for graphing functions using a TI-83/84 type of calculator. All those techniques can be used to analyze the graphs of nonlinear functions. We encourage you to review those sections before attempting to solve the Technology Exercises for this section.

Check Your Understanding 1.1

Find the slopes of the following lines.

1. The line whose equation is $x = 3y - 7$

2. The line going through the points $(2, 5)$ and $(2, 8)$

EXERCISES 1.1

Find the slopes and y-intercepts of the following lines.

1. $y = 3 - 7x$

2. $y = \dfrac{3x + 1}{5}$

3. $x = 2y - 3$

4. $y = 6$

5. $y = \dfrac{x}{7} - 5$

6. $4x + 9y = -1$

Find an equation of the given line.

7. Slope is -1; $(7, 1)$ on line. $y - 1 = -(x - 7)$

8. Slope is 2; $(1, -2)$ on line. $y + 2 = 2(x - 1)$

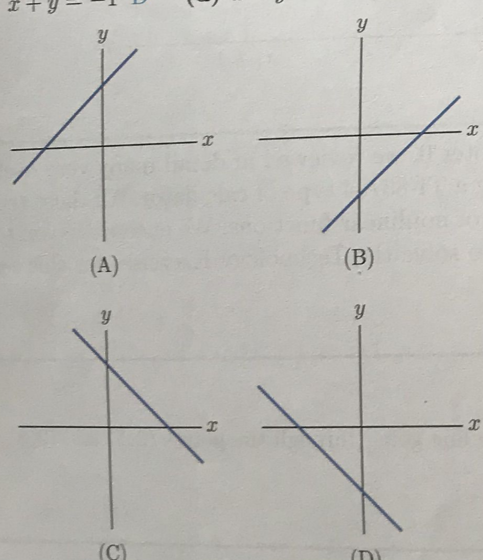
9. Slope is $\frac{1}{2}$; $(2, 1)$ on line. $y - 1 = 1/2(x - 2)$

* indicates answers that are in the back of the book. 1. $m = -7$, $b = 3$ 2. $m = 3/5$, $b = 1/5$ 3. $m = 1/2$, $b = 3/2$ 4. $m = 0$, $b = 6$ 5. $m = 1/7$, $b = -5$ 6. $m = -4/9$, $b = -1/9$

10. Slope is $\frac{7}{3}$; $(\frac{1}{4}, -\frac{2}{5})$ on line. $y+2/5 = 7/3(x-1/4)$
11. $(\frac{5}{7}, 5)$ and $(-\frac{5}{7}, -4)$ on line $y-5 = 63/10(x-5/7)$
12. $(\frac{1}{2}, 1)$ and $(1, 4)$ on line $y-4 = 6(x-1)$
13. $(0, 0)$ and $(1, 0)$ on line $y=0$
14. $(-\frac{1}{2}, -\frac{1}{7})$ and $(\frac{2}{3}, 1)$ on line $y-1 = 48/49(x-2/3)$
15. Horizontal through $(2, 9)$ $y=9$
16. x-intercept is 1; y-intercept is -3. $x - y/3 = 1$ or $y = 3x-3$
17. x-intercept is $-\pi$; y-intercept is 1.
18. Slope is 2; x-intercept is -3. $y = 2(x+3)$
19. Slope is -2; x-intercept is -2. $y = -2x-4$
20. Horizontal through $(\sqrt{7}, 2)$ $y=2$
21. Parallel to $y = x$; $(2, 0)$ on line $y = x-2$
22. Parallel to $x + 2y = 0$; $(1, 2)$ on line $y-2 = -1/2(x-1)$
23. Parallel to $y = 3x + 7$; x-intercept is 2 $y = 3x-6$
24. Parallel to $y - x = 13$; y-intercept is 0 $y = x$
25. Perpendicular to $y + x = 0$; $(2, 0)$ on line $y = x-2$
26. Perpendicular to $y = -5x + 1$; $(1, 5)$ on line

In Exercises 27–30, we specify a line by giving the slope and one point on the line. Start at the given point and use Slope Property 1 to sketch the graph of the line.

27. $m = 1$, $(1, 0)$ on line*
28. $m = \frac{1}{2}$, $(-1, 1)$ on line*
29. $m = -\frac{1}{3}$, $(1, -1)$ on line*
30. $m = 0$, $(0, 2)$ on line*
31. Each of lines (A), (B), (C), and (D) in the figure is the graph of one of the equations (a), (b), (c), and (d). Match each equation with its graph.
(a) $x + y = 1$ C
(b) $x - y = 1$ B
(c) $x + y = -1$ D
(d) $x - y = -1$ A



32. The line through the points $(-1, 2)$ and $(3, b)$ is parallel to $x + 2y = 0$. Find b. $b=0$

In Exercises 33–36, refer to a line of slope m. If you begin at a point on the line and move h units in the x-direction, how many units must you move in the y-direction to return to the line?

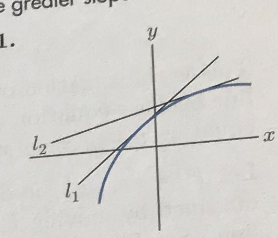
33. $m = \frac{1}{3}$, $h = 3$ 1
34. $m = 2$, $h = \frac{1}{2}$ 1
35. $m = -3$, $h = .25$ $-3/4$
36. $m = \frac{2}{3}$, $h = \frac{1}{2}$ 1/3

In Exercises 37 and 38, we specify a line by giving the slope and one point on the line. We give the first coordinate of some points on the line. Without deriving the equation of the line, find the second coordinate of each point.

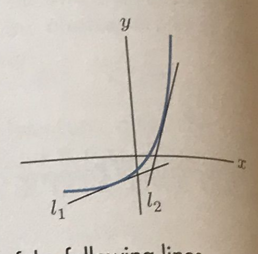
37. Slope is 2, $(1, 3)$ on line; $(2, \ )$; $(3, \ )$; $(0, \ )$.
38. Slope is -3, $(2, 2)$ on line; $(3, \ )$; $(4, \ )$; $(1, \ )$.
39. If $f(x)$ is a linear function, $f(1) = 0$, and $f(2) = 1$, what is $f(3)$? $f(3) = 2$
40. Is the line through the points $(3, 4)$ and $(-1, 2)$ parallel to the line $2x + 3y = 0$? Justify your answer.*

For each pair of lines in the following figures, determine the one with the greater slope.

41.



42.



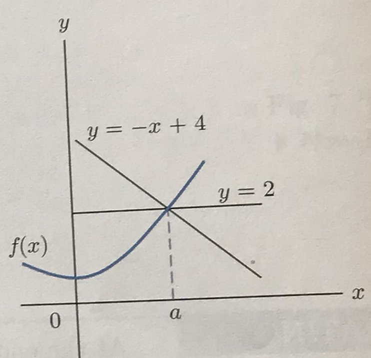
l_1 l_2

Find the equation and sketch the graph of the following lines.

43. With slope -2 and y-intercept $(0, -1)$*
44. With slope $\frac{1}{3}$ and y-intercept $(0, 1)$*

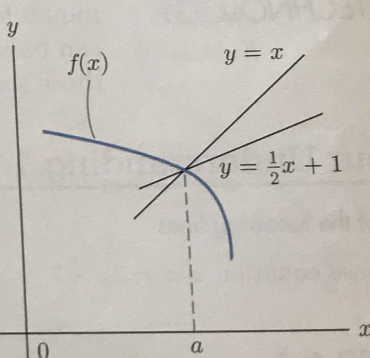
In Exercises 45 and 46, two lines intersect the graph of a function $y = f(x)$ as shown in the figure. Find a and $f(a)$.

45.



$a = 2$, $f(2) =$

46.



$a = 2$, $f(a) =$

47. **Marginal Cost** Let $C(x) = 12x + 1100$ denote the total cost (in dollars) of manufacturing x units of a certain commodity per day.
(a) What is the total cost if the production is set at 10 units per day?*
(b) What is the marginal cost?*
(c) Use (b) to determine the additional cost of raising the daily production level from 10 to 11 units.*

17. $-x/\pi + y = 1$ or $y = x/\pi + 1$ 26. $y - 5 = 1/5(x-1)$ 37. $(2,5)$; $(3,7)$; $(0,1)$ 38. $(3,-1)$; $(4,-4)$; $(1,5)$

48. Refer to Exercise 47. Use the formula for $C(x)$ to show directly that $C(x+1) - C(x) = 12$. Interpret your result as it pertains to the marginal cost.

49. Price of Gasoline The price of 1 gallon of unleaded gasoline at the pump reached \$4.12 on January 1, 2012, and continued to rise at the rate of 6 cents per month for the next 9 months. Express the price of 1 gallon of unleaded gasoline as a function of time for the period starting January 1, 2012. What was the price of 15 gallons of gasoline on April 1, 2012? On September 1, 2012? *

50. Impact of Mad Cow Disease on Canadian Beef Exports The discovery of one case of bovine spongiform encephalopathy, or mad cow disease, in May 2003 in Canada led to an immediate ban on all Canadian beef exports. At the beginning of September 2003, the ban was lifted, and exports of Canadian beef rose at a steady rate of \$42.5 million per month. Express the value of the monthly exports of Canadian beef as a function of time for the period starting the first day of September 2003. What was the value of the monthly exports at the end of December 2003, when presumably the exports regained their normal level? (*Source: International Trade Division Statistics, Canada.*)

51. Cost of Shipping and Handling An online bookstore charges \$5 plus 3% of the purchase price of books for shipping and handling. Find a function $C(x)$ that expresses the shipping and handling charge for a book order that costs x dollars. $C(x) = .03x + 5$

52. Quit Ratio In industry, the relationship between wages and the quit ratio of employees is defined to be the percentage of employees that quit within 1 year of employment. The quit ratio of a large restaurant chain that paid its employees the minimum hourly wage (\$6.55 per hour) was .2 or 20 employees per 100. When the company raised the hourly wage to \$8, the quit ratio dropped to .18, or 18 employees per 100.

(a) Assuming a linear relationship between the quit ratio $Q(x)$ and the hourly wage x, find an expression for $Q(x)$. $Q(x) = -2/145x + 421/1450$

(b) What should the hourly wage be for the quit ratio to drop to 10 employees per 100? \$13.80

53. Price Affects Sales When the owner of a gas station sets the price of 1 gallon of unleaded gasoline at \$4.10, he can sell approximately 1500 gallons per day. When he sets the price at \$4.25 per gallon, he can sell approximately 1250 gallons per day. Let $G(x)$ denote the number of gallons of unleaded gasoline sold per day when the price is x dollars. Assume that $G(x)$ is a linear function of x. Approximately how many gallons will be sold per day if the price is set at \$4.35 per gallon? *

54. Refer to Exercise 53. Where should the owner set the price if he wants to sell 2200 gallons per day? \$3.68

55. Marginal Cost Analysis A company manufactures and sells fishing rods. The company has a fixed cost of \$1500 per day and a total cost of \$2200 per day when the production is set at 100 rods per day. Assume that the total cost $C(x)$ is linearly related to the daily production level x.

(a) Express the total cost as a function of the daily production level. $C(x) = 7x + 1500$

(b) What is the marginal cost at production level $x = 100$? Marginal cost is \$7 per rod.

(c) What is the additional cost of raising the daily production level from 100 to 101 rods? Answer this question in two different ways: (1) by using the marginal cost and (2) by computing $C(101) - C(100)$. \$7

56. Interpreting the Slope and y-Intercept A salesperson's weekly pay depends on the volume of sales. If she sells x units of goods, her pay is $y = 5x + \$60$. Give an interpretation of the slope and the y-intercept of this straight line. slope = price per sale y-int = base commission

57. Interpreting the Slope and y-Intercept The demand equation for a monopolist is $y = -.02x + 7$, where x is the number of units produced and y is the price. That is, to sell x units of goods, the price must be $y = -.02x + \$7$. Interpret the slope and y-intercept of this line. *

58. Converting Fahrenheit to Celsius Temperatures of 32°F and 212°F correspond to temperatures of 0°C and 100°C. The linear equation $y = mx + b$ converts Fahrenheit temperatures to Celsius temperatures. Find m and b. What is the Celsius equivalent of 98.6°F?

59. Intravenous Injection A drug is administered to a patient through an IV (intravenous) injection at the rate of 6 milliliters (mL) per minute. Assuming that the patient's body already contained 1.5 mL of this drug at the beginning of the infusion, find an expression for the amount of the drug in the body x minutes from the start of the infusion. *

60. Refer to Exercise 59. If the patient's body eliminates the drug at the rate of 2 mL per hour, find an expression for the amount of the drug in the body x minutes from the start of the infusion. $y = 179/30x + 1.5$

61. Diver's Ascent After inspecting a sunken ship at a depth of 212 feet, a diver starts her slow ascent to the surface of the ocean, rising at the rate of 2 feet per second. Find $y(t)$, the depth of the diver, measured in feet from the ocean's surface, as a function of time t (in seconds). *

62. Diver's Ascent The diver in the previous exercise is supposed to stop for 5 minutes and decompress at 150 feet depth. Assuming that the diver will continue her ascent, after decompressing, at the same rate of 2 feet per second, find $y(t)$ in this case and determine how long it will take the diver to reach the surface of the ocean. *

63. Sale of T-Shirts A T-shirt shop owner has a fixed cost of \$230 and a marginal cost of \$7 per T-shirt to manufacture x T-shirts per day. Let $C(x)$ denote the cost to manufacture x T-shirts per day.

(a) Find $C(x)$. $C(x) = 7x + 230$ dollars

(b) If the shop owner decides to sell the T-shirts at \$12 each, find $R(x)$, the total revenue from selling x T-shirts per day. $R(x) = 12x$ dollars

64. Break-Even In order for a business to break even, revenue has to equal cost. Determine the minimum number of T-shirts that should be sold in the previous exercise to break even. Break-even when $x = 46$

65. Prove Slope Property 4 of straight lines. [*Hint:* If $y = mx + b$ and $y = m'x + b'$ are two lines, they have a point in common if, and only if, the equation $mx + b = m'x + b'$ has a solution x.]

48. $C(x+1) - C(x) = 12(x+1) + 1100 - 12x - 1100 = 12$ 50. $y = 42.5x$, $y(4) = \$170$ million. 58. $m = 5/9$, $b = -160/9$, $98.6°F = 37°C$