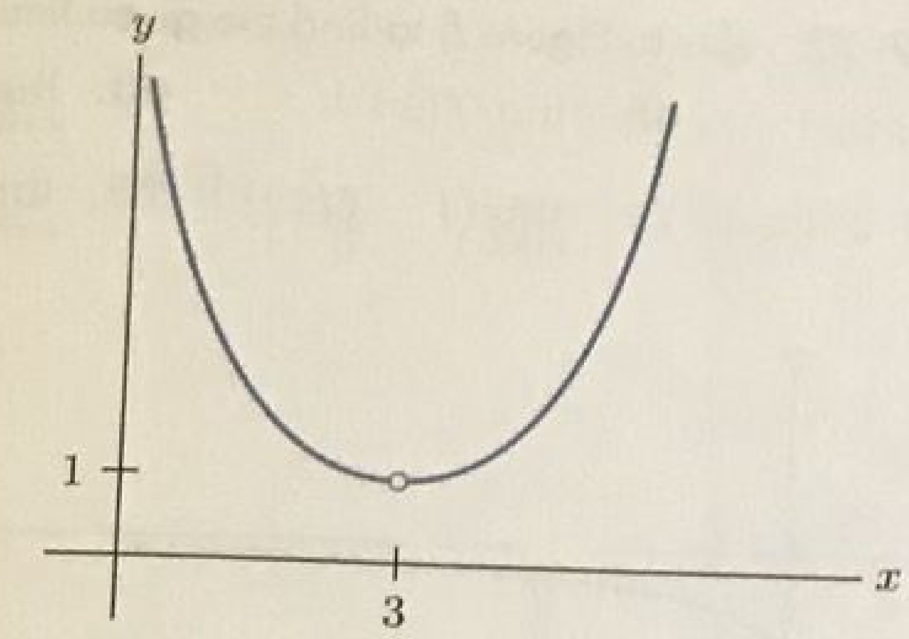
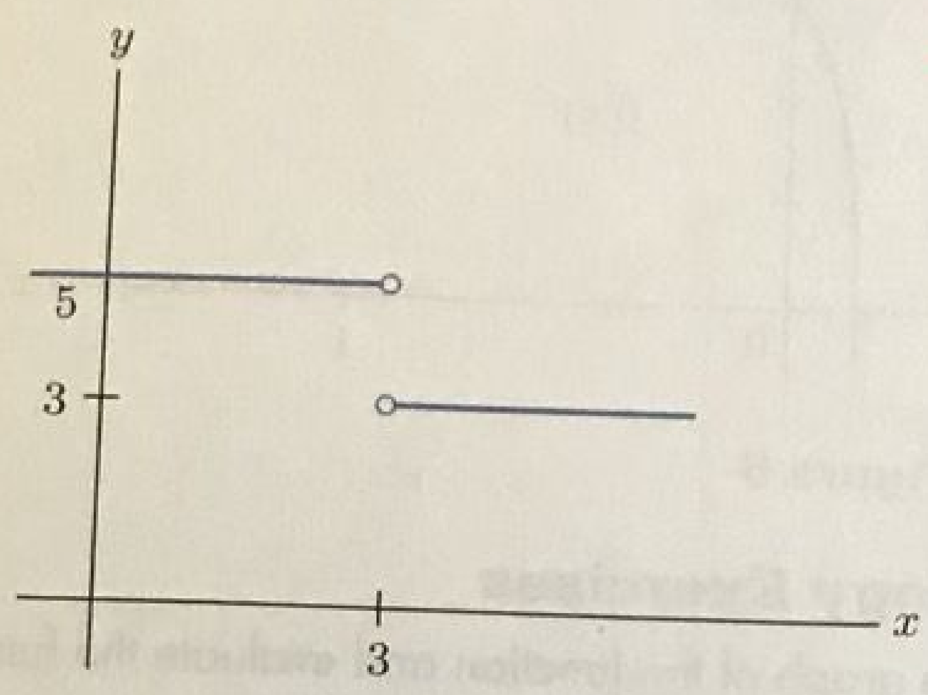
3.

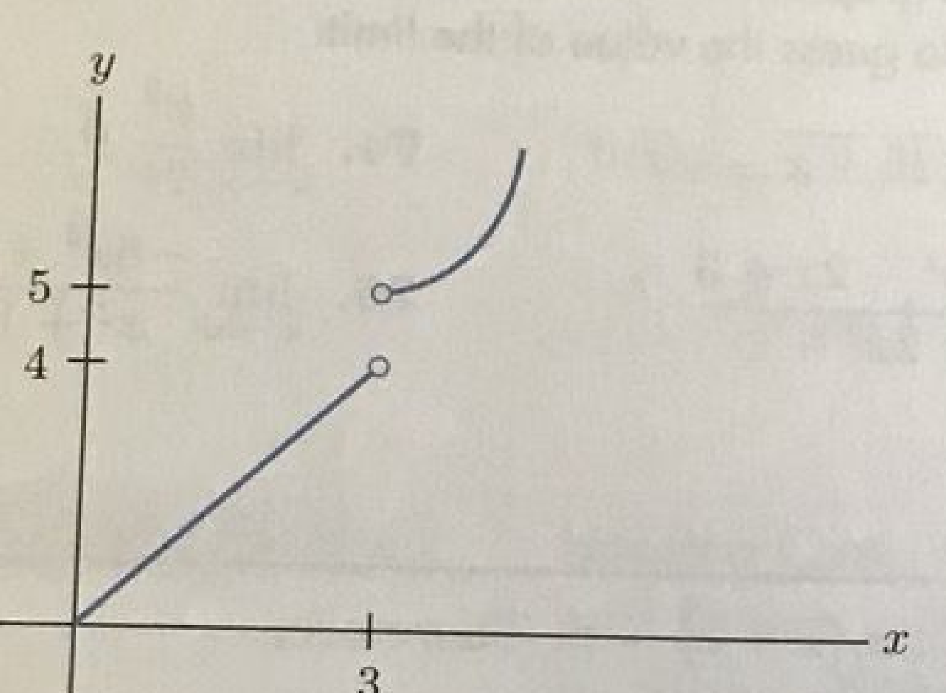


1

4.

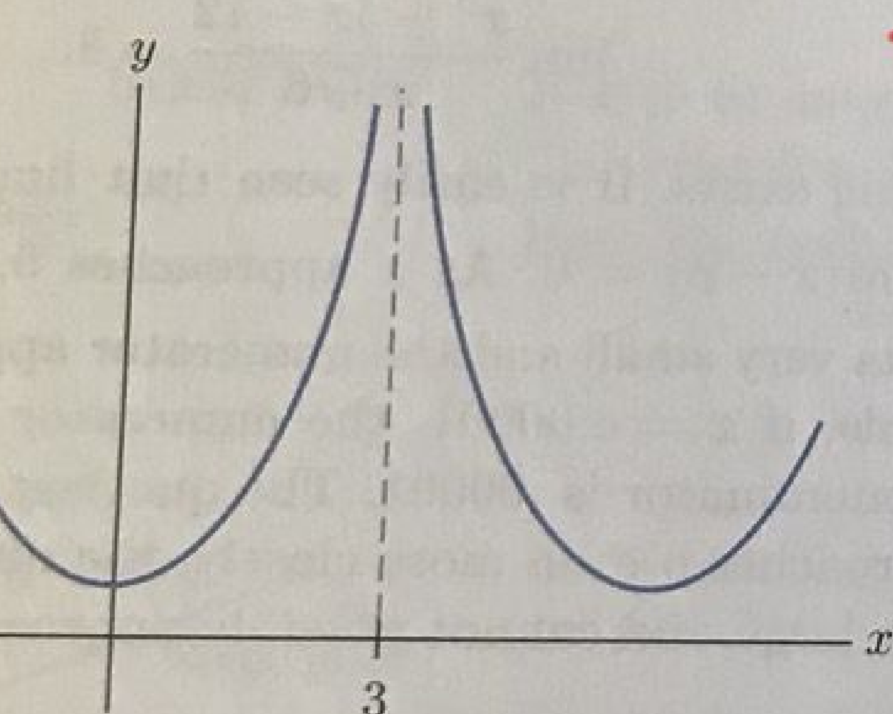


5.



No limit

6.



Determine which of the following limits exist. Compute the limits that exist.

7. $\lim_{x\to 1}(1-6x)$ -5

8. $\lim_{x\to 2}\frac{x}{x-2}$ No limit

9. $\lim_{x\to 3}\sqrt{x^2+16}$ 5

10. $\lim_{x\to 4}(x^3-7)$

11. $\lim_{x\to 5}\frac{x^2+1}{5+x}$ 13/5

12. $\lim_{x\to 6}\left(\sqrt{6x}+3x-\frac{1}{x}\right)(x^2-4)$ 2288/3

13. $\lim_{x\to 7}(x+\sqrt{x-6})(x^2-2x+1)$ 288

14. $\lim_{x\to 8}\frac{\sqrt{5x-4}-1}{3x^2+2}$ 5/194

15. $\lim_{x\to -5}\frac{\sqrt{x^2-5x-36}}{8-3x}$ $\sqrt{14}/23$

16. $\lim_{x\to 10}(2x^2-15x-50)^{20}$ 0

17. $\lim_{x\to 0}\frac{x^2+3x}{x}$ 3

18. $\lim_{x\to 1}\frac{x^2-1}{x-1}$ 2

19. $\lim_{x\to 2}\frac{-2x^2+4x}{x-2}$ -4

20. $\lim_{x\to 3}\frac{x^2-x-6}{x-3}$ 5

21. $\lim_{x\to 4}\frac{x^2-16}{4-x}$ -8

22. $\lim_{x\to 5}\frac{2x-10}{x^2-25}$

23. $\lim_{x\to 6}\frac{x^2-6x}{x^2-5x-6}$ 6/7

24. $\lim_{x\to 7}\frac{x^3-2x^2+3x}{x^2}$ 38/7

25. $\lim_{x\to 8}\frac{x^2+64}{x-8}$ No limit

26. $\lim_{x\to 9}\frac{1}{(x-9)^2}$ No limit

27. Compute the limits that exist, given that

$$\lim_{x\to 0}f(x)=-\frac{1}{2}\quad\text{and}\quad\lim_{x\to 0}g(x)=\frac{1}{2}.$$

(a) $\lim_{x\to 0}(f(x)+g(x))$ (b) $\lim_{x\to 0}(f(x)-2g(x))$

(c) $\lim_{x\to 0}f(x)\cdot g(x)$ (d) $\lim_{x\to 0}\frac{f(x)}{g(x)}$

28. Use the limit definition of the derivative to show that if $f(x)=mx+b$, then $f'(x)=m$.

Use limits to compute the following derivatives.

29. $f'(3)$, where $f(x)=x^2+1$ 6

30. $f'(2)$, where $f(x)=x^3$ 12

31. $f'(0)$, where $f(x)=x^3+3x+1$ 3

32. $f'(0)$, where $f(x)=x^2+2x+2$ 2

In Exercises 33–36, apply the three-step method to compute $f'(x)$ for the given function. Follow the steps that we used in Example 6. Make sure to simplify the difference quotient as much as possible before taking limits.

33. $f(x)=x^2+1$

34. $f(x)=-x^2+2$

35. $f(x)=x^3-1$

36. $f(x)=-3x^2+1$

In Exercises 37–48, use limits to compute $f'(x)$. [*Hint:* In Exercises 45–48, use the rationalization trick of Example 8.]

37. $f(x)=3x+1$

38. $f(x)=-x+11$ $f'(x)=-1$

39. $f(x)=x+\frac{1}{x}$

40. $f(x)=\frac{1}{x^2}$ $f'(x)=-2/x^3$

41. $f(x)=\frac{x}{x+1}$

42. $f(x)=-1+\frac{2}{x-2}$

43. $f(x)=\frac{1}{x^2+1}$

44. $f(x)=\frac{x}{x+2}$ $f'(x)=2/(x+2)^2$

45. $f(x)=\sqrt{x+2}$

46. $f(x)=\sqrt{x^2+1}$ $f'(x)=x/\sqrt{x^2+1}$

47. $f(x)=\frac{1}{\sqrt{x}}$

48. $f(x)=x\sqrt{x}$ $f'(x)=3/2\sqrt{x}$

Each limit in Exercises 49–54 is a definition of $f'(a)$. Determine the function $f(x)$ and the value of a.

49. $\lim_{h\to 0}\frac{(1+h)^2-1}{h}$ $f(x)=x^2;\ a=1$

50. $\lim_{h\to 0}\frac{(2+h)^3-8}{h}$ $f(x)=x^3,\ a=2$

51. $\lim_{h\to 0}\frac{\frac{1}{10+h}-.1}{h}$ $f(x)=1/x;\ a=10$

52. $\lim_{h\to 0}\frac{(64+h)^{1/3}-4}{h}$

53. $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}$ $f(x)=\sqrt{x};\ a=9$

54. $\lim_{h\to 0}\frac{(1+h)^{-1/2}-1}{h}$

28. $f(x+h)=mx+mh+b$, $f(x+h)-f(x)=mh$, $f(x+h)-f(x)/h=m$, $\lim_{h\to 0}m=m$ 42. $f'(x)=-2/(x-2)^2$ 52. $f(x)=\sqrt[3]{x}$, $a=64$ 54. $f(x)=1/\sqrt{x}$, $a=1$

In Exercises 55–60, match the given limit with a derivative and then find the limit by computing the derivative.

55. $\lim_{h\to 0} \frac{(h+2)^2-4}{h}$

56. $\lim_{h\to 0} \frac{(h-1)^3+1}{h}$ $\frac{d}{dx}x^3\big|_{x=-1} = 3$

57. $\lim_{h\to 0} \frac{\sqrt{h+2}-\sqrt{2}}{h}$

58. $\lim_{h\to 0} \frac{\sqrt{h+4}-2}{h}$ $\frac{d}{dx}\sqrt{x}\big|_{x=4} = \frac{1}{4}$

59. $\lim_{h\to 0} \frac{(8+h)^{1/3}-2}{h}$

60. $\lim_{h\to 0} \frac{1}{h}\left[\frac{1}{h+1}-1\right]$ $\frac{d}{dx}\frac{1}{x}\big|_{x=1} = -1$

Compute the following limits.

61. $\lim_{x\to\infty} \frac{1}{x^2}$ 0

62. $\lim_{x\to-\infty} \frac{1}{x^2}$ 0

63. $\lim_{x\to\infty} \frac{5x+3}{3x-2}$ 5/3

64. $\lim_{x\to\infty} \frac{1}{x-8}$

65. $\lim_{x\to\infty} \frac{10x+100}{x^2-30}$ 0

66. $\lim_{x\to\infty} \frac{x^2+x}{x^2-1}$

55. Take $f(x)=x^2$; then, the given limit is $f'(2)=4$.
57. Take $f(x)=\sqrt{x}$; then, the given limit is $f'(2)=1/2\sqrt{2}$.
59. Take $f(x)=x^{1/3}$; then, the given limit is $f'(8)=1/12$.

In Exercises 67–72, refer to Figure 5 to find the given limit.

67. $\lim_{x\to 0} f(x)$ 3/4

68. $\lim_{x\to\infty} f(x)$ 1

69. $\lim_{x\to 0} x f(x)$ 0

70. $\lim_{x\to\infty} (1+2f(x))$ 3

71. $\lim_{x\to\infty} (1-f(x))$ 0

72. $\lim_{x\to 0} [f(x)]^2$ 9/16

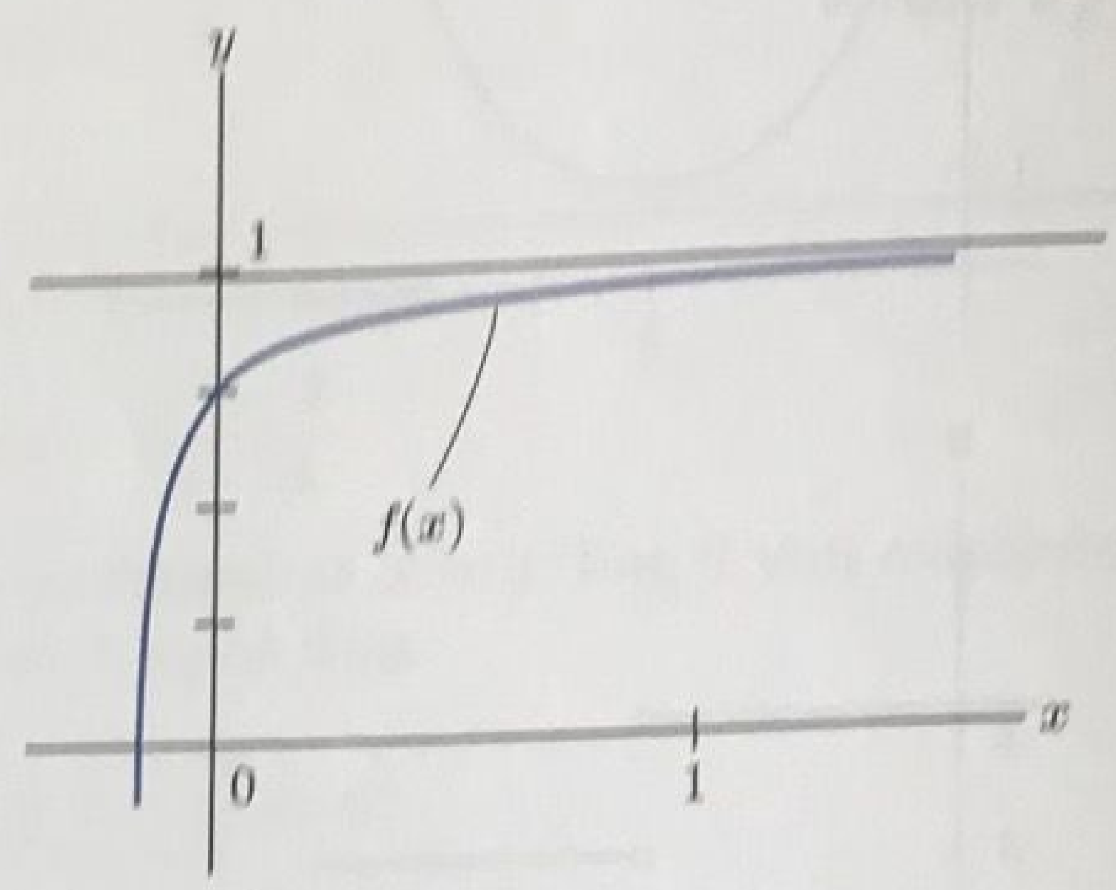


Figure 5

Technology Exercises

Examine the graph of the function and evaluate the function at large values of x to guess the value of the limit.

73. $\lim_{x\to\infty} \sqrt{25+x}-\sqrt{x}$ 0

74. $\lim_{x\to\infty} \frac{x^2}{2^x}$ 0

75. $\lim_{x\to\infty} \frac{x^2-2x+3}{2x^2+1}$ .5

76. $\lim_{x\to\infty} \frac{-8x^2+1}{x^2+1}$ −8

Solutions to Check Your Understanding 1.4

1. The function under consideration is a rational function. Since the denominator has value 0 at $x=6$, we cannot immediately determine the limit by just evaluating the function at $x=6$. Also,

$$\lim_{x\to 6}(x-6)=0.$$

Since the function in the denominator has the limit zero, we cannot apply Limit Theorem VI. However, since the definition of limit considers only values of x different from 6, the quotient can be simplified by factoring and canceling:

$$\frac{x^2-4x-12}{x-6} = \frac{(x+2)(x-6)}{(x-6)} = x+2 \quad \text{for } x \neq 6.$$

Now, $\lim_{x\to 6}(x+2)=8$. Therefore,

$$\lim_{x\to 6}\frac{x^2-4x-12}{x-6}=8.$$

2. No limit exists. It is easily seen that $\lim_{x\to 6}(4x+12)=36$ and $\lim_{x\to 6}(x-6)=0$. As x approaches 6, the denominator gets very small and the numerator approaches 36. For example, if $x=6.00001$, the numerator is 36.00004 and the denominator is .00001. The quotient is 3,600,004. As x approaches 6 even more closely, the quotient gets arbitrarily large and cannot possibly approach a limit.

1.5 Differentiability and Continuity

In the preceding section, we defined differentiability of $f(x)$ at $x=a$ in terms of a limit. If this limit does not exist, then we say that $f(x)$ is *nondifferentiable* at $x=a$. Geometrically, the nondifferentiability of $f(x)$ at $x=a$ can manifest itself in several different ways. First, the graph of $f(x)$ could have no tangent line at $x=a$. Second, the graph could have a vertical tangent line at $x=a$. (Recall that slope is not defined for vertical lines.) Some of the various geometric possibilitie[illegible] illustrated in Fig. 1.

The following example illustrates how nondifferenti[illegible] fu[illegible] an aris[illegible] practice.