Midterm 2 – Math 241

Friday, October 19, 2018

This is a closed-book exam. No calculators allowed.
Justify your answers to obtain full credit (and partial credit, too).
You have 50 minutes.
This exam consists of 6 questions.
Please verify that you have all pages.

Name:_____ **ID#:**_____

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1. (*20 points*) Differentiate the following functions. You do not need to simplify your answers.

(a) $y = 3x \sin x$

(b) $y = \tan(x^2 + 1)$

(c) $y = \dfrac{x+1}{x^2+2}$

(d) $y = \dfrac{(x+1)^{(3/2)}}{2x^3 + 4x^2 + 7}$

2. (*15 points*) Show that $f(x) = 2x^3 + 3x^2 + 6x + 1$ has exactly one real root in $[-1, 0]$. Be sure to state and explain any theorems that you use.

3. (*10 points*) Find an equation of the tangent line to $x^2 + xy + y^2 = 3$ at $(1, 1)$

4. (*25 points*) Let $f(x) = x^3 + 3x^2$

(a) Find the (open) intervals where f is increasing and where f is decreasing.

(b) Find all relative extrema (both x and y coordinates). Indicate whether it is a relative maximum or relative minimum.

(c) Find the (open) intervals where f is concave up and where f is concave down

(d) Find all inflection point(s) (both x and y coordinates)

(e) Using the information from parts (a)-(d), graph the function. Label all relative extrema and inflection point(s).

5. (*20 points*) An ecologist is conducting a research project on breeding pheasants in captivity. She first must construct suitable pens. She wants a rectangular area with two additional fences across its width, as shown in the sketch. Find the **dimensions** of the pen that has the maximum area she can enclose with 3600 m of fencing.

6. (*10 points*) A spherical snowball is placed in the sun. The sun melts the snowball so that its radius **decreases** 1/4 in. per hour. Find the rate of change of the volume with respect to time at the instant the radius is 4 in. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Final Score

	Score	Out of
Question 1		20
Question 2		15
Question 3		10
Question 4		25
Question 5		20
Question 6		10
Total		100