Math 241 Midterm 1 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, quizzes, and homework problems.

(1) Calculate the following limits.

(a) $\lim\limits_{t \to 1} \dfrac{t^2 + t - 2}{t^2 - 1}$

(b) $\lim\limits_{x \to 2} \dfrac{\sqrt{x^2 + 12} - 4}{x - 2}$

(c) $\lim\limits_{x \to \infty} \dfrac{2\sqrt{x} + x^{-1}}{3x - 7}$

(d) $\lim\limits_{x \to \infty} \dfrac{7x^3}{x^3 - 3x^2 + 6x}$

(e) $\lim\limits_{x \to 0^-} \dfrac{x^2}{2} - \dfrac{1}{x}$

(f) $\lim\limits_{x \to 3} 4x^3$

(g) $\lim\limits_{x \to 0} \dfrac{1}{x^3 - 1} + 1$

(h) $\lim\limits_{x \to 1} \dfrac{3x - 4}{x^2 + x + 1}$

(i) $\lim\limits_{x \to -1} \dfrac{x^2 + x - 2}{x^3 + 1}$

(j) $\lim\limits_{x \to -1^-} \dfrac{|x + 1|}{x + 1}$

(k) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

(l) $\lim\limits_{x \to -3} \sqrt{x - 5}$

(m) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 4x^2 + 5x}{17x^2 + 1}$

(n) $\lim\limits_{x \to -\infty} \dfrac{3x^2 + 4}{x + 7}$

(o) $\lim\limits_{t \to 2^-} \dfrac{t + 2}{t - 2}$

(p) $\lim\limits_{x \to 0} \dfrac{\sin x}{3x}$

(q) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x} + 3x^{-2}}{4 - 2x}$

(r) $\lim\limits_{x \to 0} x \cos\left(\dfrac{1}{x}\right)$

(s) $\lim\limits_{x \to \infty} \dfrac{\sin x}{x}$

(2) Describe on which intervals the following functions are continuous (show your work):

(a) $y = \dfrac{\sin x}{x - 2}$

(b) $f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & 2 \le x < 4 \\ 3, & x \ge 4 \end{cases}$

(c) $f(x) = \begin{cases} 1 - x^2 & x < -1 \\ 1 + x & -1 \le x \le 1 \\ -3 & x > 1 \end{cases}$

(3) Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$

(4) Use the definition of the derivative (limit definition) to find the derivatives of the following:

    (a) $f(x) = \sqrt{x}$

    (b) $f(x) = x^2 - x$

    (c) $f(x) = \frac{1}{x}$

(5) Consider the function $f(x) = 5 - x^2$.

    (a) Find the equation for the secant line to the graph of $f(x)$ that passes through the points $(1, 4)$ and $(2, 1)$.

    (b) Find $f'(x)$ using the definition of a derivative.

    (c) Find the equation for the tangent line to the graph of $f(x)$ at the point $(1, 4)$.

    (d) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2, 1)$.

(6) A particle is moving along the x-axis. Its position at time t is given by the function $s(t) = -2t^2 + 5t - 2$.

    (a) Find the particle's average velocity v_{av} between $t = 1$ and $t = 4$.

    (b) Find the particle's instantaneous velocity at $t = 1$.