Math 241 Midterm 2 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, quizzes, and homework problems.

(1) Find the first derivatives of the following:

   (a) $y = 6x^2 - 10x - 5x^{-2}$

   (b) $y = x^2 \sin x + 2x \cos x - 2 \sin x$

   (c) $h(x) = x \tan(2\sqrt{x}) + 7$

   (d) $y = \dfrac{\cot x}{1 + \cot x}$

   (e) $y = \left(1 - \dfrac{x}{7}\right)^{-7}$

(2) If a particle's motion is given by the equation $s(t) = 4t^3 - 10t^2 + 5$, find its velocity and acceleration as functions of t. What is its speed at $t = 1$

(3) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ if $2\sqrt{y} = x - y$

(4) Find $\dfrac{dy}{dx}$ if $y \sin\left(\dfrac{1}{y}\right) = 1 - xy$.

(5) Find equations for the tangent and normal lines to $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-1, 0)$.

(6) The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

(7) A rectangular plot of land will be bounded on one side by a river and on the other three sides by some sort of fence. With 800 m of fencing at your disposal, what is the largest area you can enclose, and what are its dimensions?

(8) You fall asleep in class and dream that you are in charge making huge square-based, open-top, rectangular boxes of steel for literally no reason. Dream-you realizes it is absolutely essential that you build one that is 500 ft^3, and this box must be made by welding steel plates together along their edges. Find the dimensions for the base and height that will make the steal box weigh as little as possible. (Note: though you are dreaming, all mathematical rules and laws of physics apply, because you're really psyched for the upcoming midterm.)

(9) A child flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from them at a rate of 25 ft/sec. How fast must they let out the string when the kite is 500 ft away from them?

(10) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radios 10 cm. What is the maximum volume?

(11) A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 10 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m?

(12) For the following functions, **a)** find the critical points, **b)** classify them as local maxima, local minima, or neither, **c)** find where the function is increasing, **d)** find where the function is concave up, and **e)** sketch the graph.

   (a) $y = x^4 - 2x^2$
   (b) $y = x^5 - 5x^4$

(13) Find the absolute maximum and minimum values of the following functions of the given intervals.

   (a) $f(x) = x^2 - 1, \ -1 \le x \le 2$
   (b) $f(x) = \sqrt[3]{x}, \ -1 \le x \le 8$