

# Math 241 Midterm 3 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, quizzes, and homework problems.

(1) Evaluate the following integrals

(a)  $\int \tan x \sec^2 x \, dx$

(b)  $\int \frac{x}{\sqrt{4x^2 + 9}} \, dx$

(c)  $\int \sec^2(5x) \, dx$

(d)  $\int x(2x + 1)^5 \, dx$

(e)  $\int_0^2 x\sqrt{x^2 + 1} \, dx$

(f)  $\int_0^{\sqrt{\pi}/2} x \sin x^2 \, dx$

(g)  $\int \left( \frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}} \right) \, dx$

(h)  $\int x^{-3}(x + 1) \, dx$

(i)  $\int_0^{\pi/3} 2 \sec^2 x \, dx$

(j)  $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2} \, dx$

(k)  $\int \frac{\sin(2t + 1)}{\cos^2(2t + 1)} \, dt$

(l)  $\int_{-1}^1 t^3(1 + t^4)^3 \, dt$

(m)  $\int_0^{\pi/6} (1 - \cos 3t) \sin 3t \, dt$

(2) Solve the initial value problem

(a)  $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}, y(4) = 0$

(b)  $\frac{ds}{dt} = 12t(3t^2 - 1)^3, s(1) = 3$

(3) The acceleration of an object is given by  $\frac{3t}{8}$  find the position given that  $v(4) = 3$  and  $s(4) = 4$ .

(4) Using 4 rectangles of equal length and the following rules find Riemann sums estimates for  $f(x) = -x^2 + 16$  from  $x = -2$  to  $x = 2$ .

(a) Left-hand endpoints

(b) Right-hand endpoints

(c) Midpoints

(5) Find  $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t \, dt$

(a) by evaluating the integral and differentiating the result.

(b) by differentiating the integral directly

(6) Find the total area between  $y = x\sqrt{4 - x^2}$  and  $y = 0$

- (7) Set up but do not evaluate the integral(s) needed to find the area between  $y = x^2 - 4$  and  $y = -x^2 - 2x$  from  $x = -3$  to  $x = 1$ .
- (8) Set up but do not evaluate the integral(s) needed to find the area between  $y = x^4 - 4x^2 + 4$  and  $y = x^2$ .
- (9) Find the volume of the solid that lies between planes perpendicular to the  $x$ -axis at  $x = 0$  and  $x = 4$ . The cross-sections perpendicular to the axis on the interval  $0 \leq x \leq 4$  are squares whose diagonals run from the parabola  $y = -\sqrt{x}$  to the parabola  $y = \sqrt{x}$ .
- (10) Find the volume of the solid that lies between planes perpendicular to the  $x$ -axis at  $x = -1$  and  $x = 1$ . The cross-sections perpendicular to the  $x$ -axis are circular disks whose diameters run from the parabola  $y = x^2$  to the parabola  $y = 2 - x^2$ .
- (11) Find the volumes of the solids generated by revolving the regions bounded by the lines and curves about the  $x$ -axis:  $y = x^2$ ,  $y = 0$ ,  $x = 2$
- (12) Find the volume of the solid generated by revolving the region the region bounded by  $y = \sqrt{x}$  and the lines  $y = 2$  and  $x = 0$  about
- (a) the  $x$ -axis
  - (b) the  $y$ -axis
  - (c) the line  $y = 2$
  - (d) the line  $x = 4$