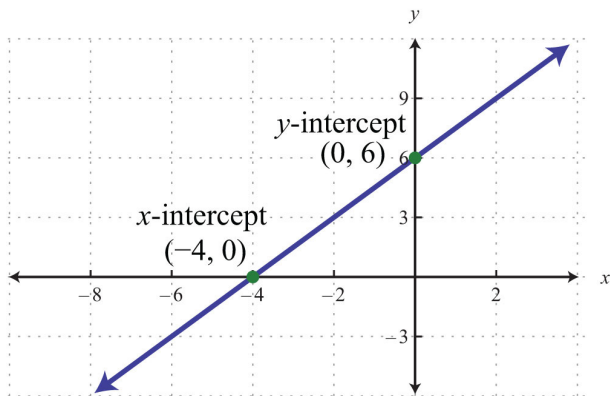


Sections 3.3-3.5 Lecture Notes

Definition: An x -intercept is the point at which a graph intersects (i.e. crosses or touches) the x -axis. A y -intercept is the point at which a graph intersects (i.e. crosses or touches) the y -axis.

Example:



Brainstorm: Given an equation of a line, how can you find the x and y intercept(s) without graphing?

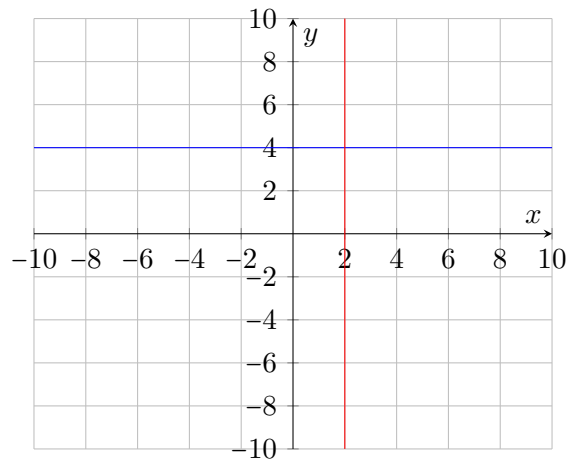
Exercise: Find the x and y intercept(s) of $3x + 4y + 12 = 0$ without graphing.

Exercise: Find the x and y intercept(s) of $f(x) = -\frac{1}{3}x - 1$ without graphing.

Definition: A horizontal line is a line that is parallel to the x -axis. A vertical line is a line that is parallel to the y -axis.

Brainstorm: What does an equation for a horizontal line look like? What does an equation for a vertical line look like?

Exercise: What are the equations of these two lines:

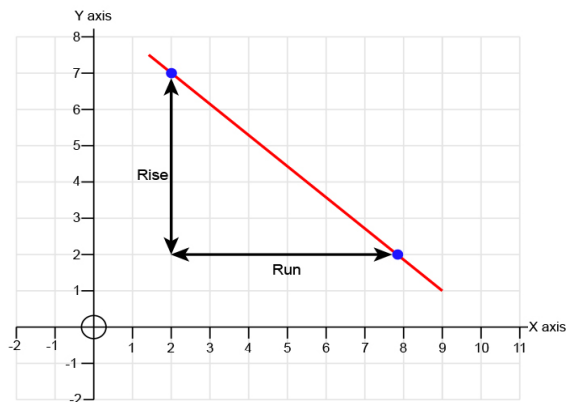


Definition: The slope of a line is the ratio of the vertical change (rise) to the horizontal change (run), between any two points on the line. It measures the steepness of the line. Typically, we use the letter m to denote slope.

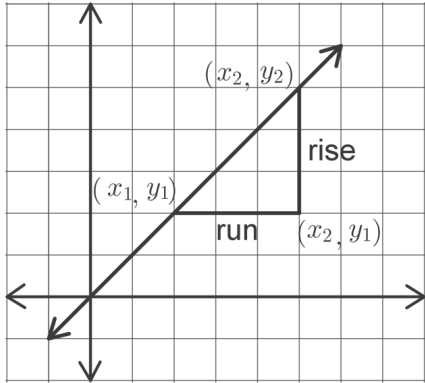
$$\text{Slope} = m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

Note: Slope can also be thought of as the line's rate of change (i.e. it tells us how much the y -value changes when the x -value increases by one).

Visual:

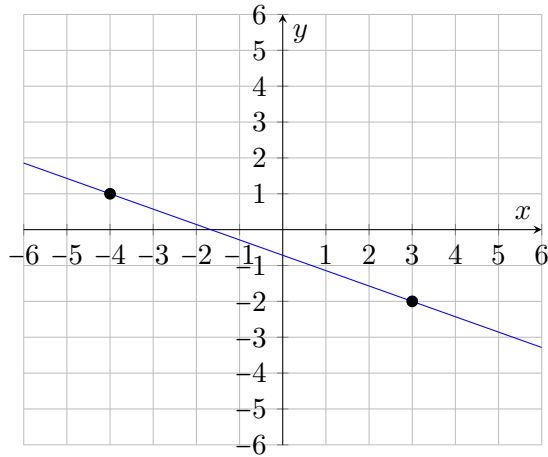


Brainstorm: Given two points (x_1, y_1) and (x_2, y_2) on a line, what equation can we use to find slope?



Exercise: Find the slope of the line passing through $(4, -2)$ and $(-1, 5)$

Exercise: Determine the slope of the following line:



Brainstorm: What general shape does a line with a positive slope have? What general shape does a line with a negative slope have? What is the slope of a horizontal line? What is the slope of a vertical line?

Definition: There are three main ways of writing the equation of a line:

- Slope-intercept form: $y = mx + b$, where m is the slope and b is the y -intercept
- Point-slope form: $y - y_1 = m(x - x_1)$ where m is the slope and the point (x_1, y_1) is on the line
- General form: $Ax + By + C = 0$, where A and B are not both 0

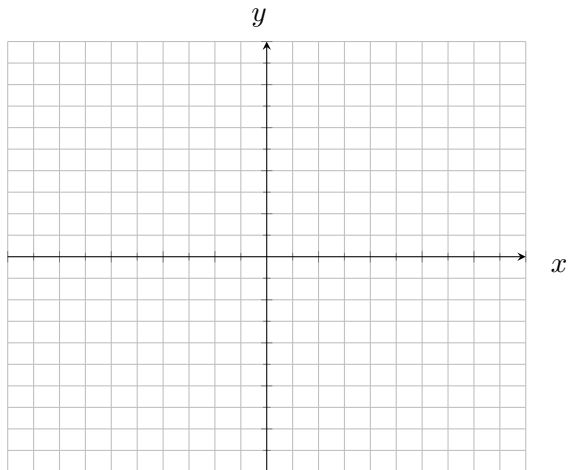
Brainstorm: How can you get from general form to slope-intercept form?

Exercise: Find the slope-intercept form of the line with equation $3x + 6y - 12 = 0$

Note: To graph a line, you only need two points! Which two points you use can depend on which you feel are the easiest to get from the equation.

Exercise: Graph the following lines:

- (1) $f(x) = \frac{3}{5}x + 1$, using the y -intercept and the slope
- (2) $3x - 2y - 6 = 0$ using the x -intercept and the y -intercept
- (3) $2y + 4x = 6$, using any method
- (4) $y = 3$, using any method



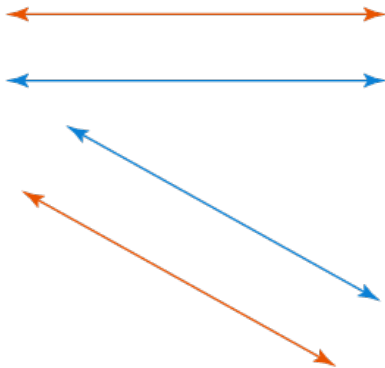
Brainstorm: Find the relationship between the equation for the slope of a line, and the point-slope form of a line.

Exercise:

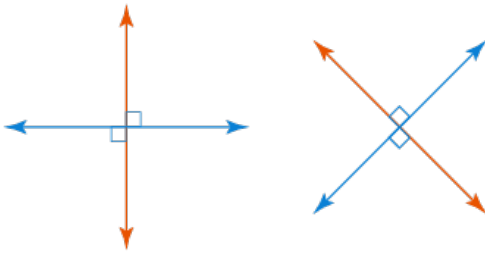
- (1) Write an equation for the line with slope 6 that passes through $(2, -5)$. Convert your equation to the other forms.
- (2) Write an equation for a line that passes through $(-4, 3)$ with slope -2 . Convert your equation to the other forms.
- (3) Write an equation for a line that passes through $(2, 3)$ and $(1, 4)$. Convert your equation to the other forms.

Brainstorm: How do the slopes of parallel lines relate to each other? How do the slopes of perpendicular lines relate to each other?

Parallel lines



Perpendicular lines



Exercise:

- (1) Write an equation of the line parallel to $y = 3x + 1$ that goes through the point $(0, 6)$.
- (2) Write an equation of the line that is perpendicular to $y = 3x + 1$ through $(2, 6)$.
- (3) Find an equation of the line perpendicular to $x + 3y - 12 = 0$ that goes through $(3, 3)$.