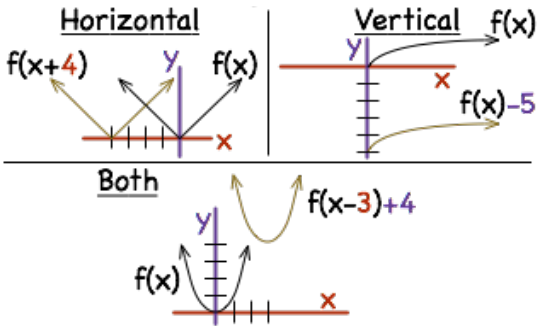
Sections 3.5 Lecture Notes

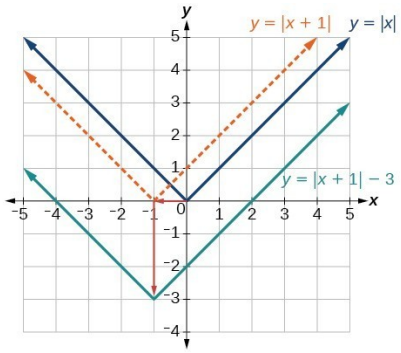
Vertical and Horizontal Shifts

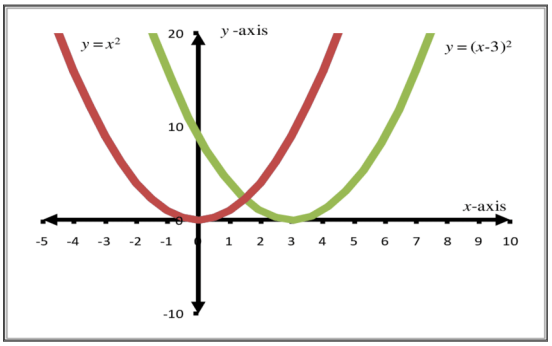
Given a parent function $y = f(x)$ and a positive number c:

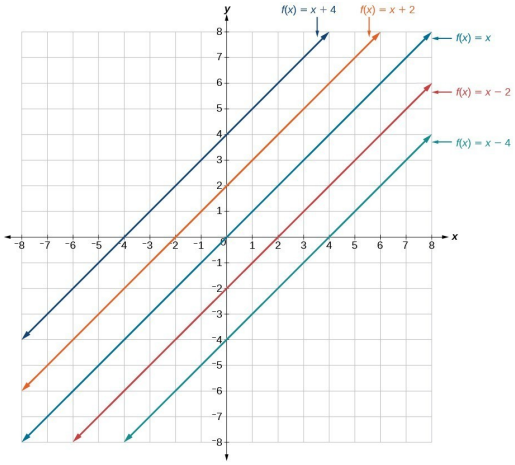
- The graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted upward c unit(s)
- The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted downward c unit(s)
- The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted left c units
- The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted right c units

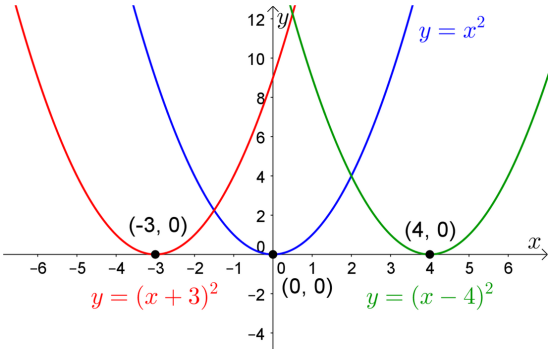
Examples:

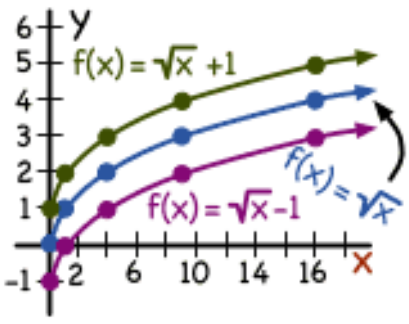










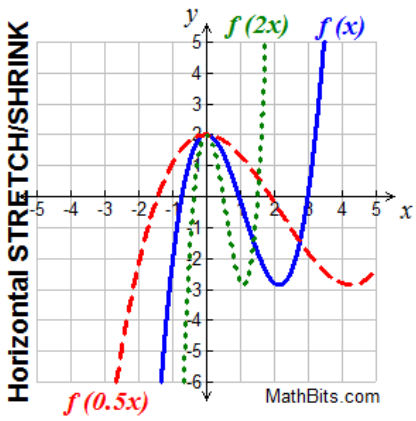


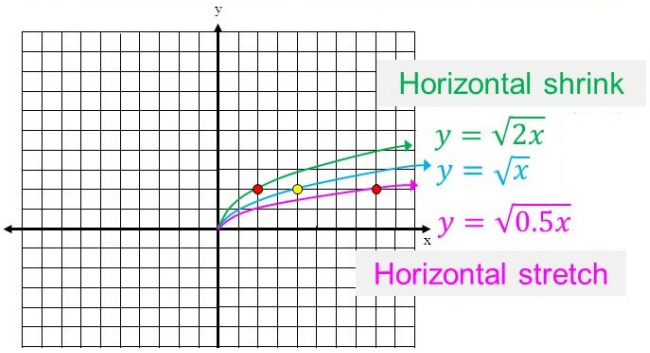
Stretching and Shrinking

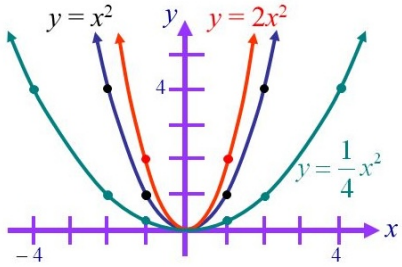
Given a parent function $y = f(x)$ and a positive number c:

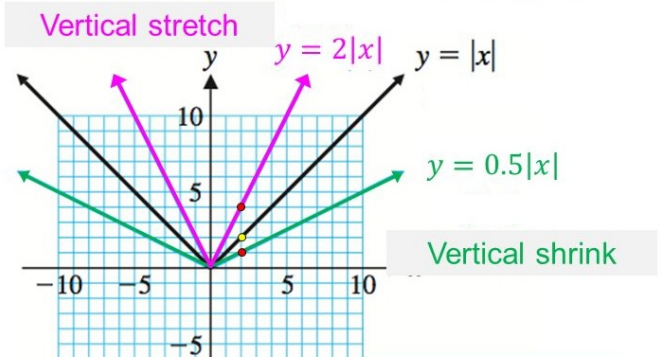
- $y = cf(x)$ is a vertical stretching if $c > 1$ (multiply each y value by c)
- $y = cf(x)$ is a vertical shrinking if $c < 1$ (multiply each y value by c)
- $y = f(cx)$ is a horizontal shrinking if $c > 1$ (divide each x value by c)
- $y = f(cx)$ is a horizontal stretching if $c < 1$ (divide each x value by c)

Examples:







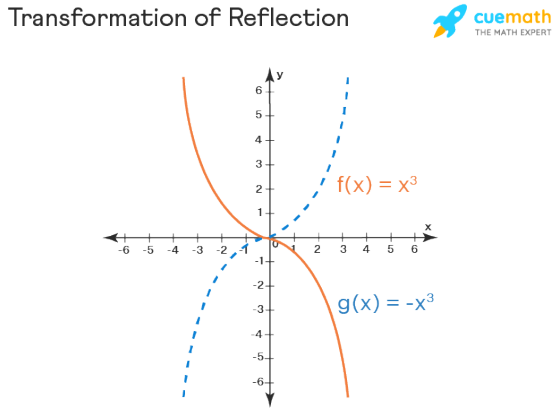


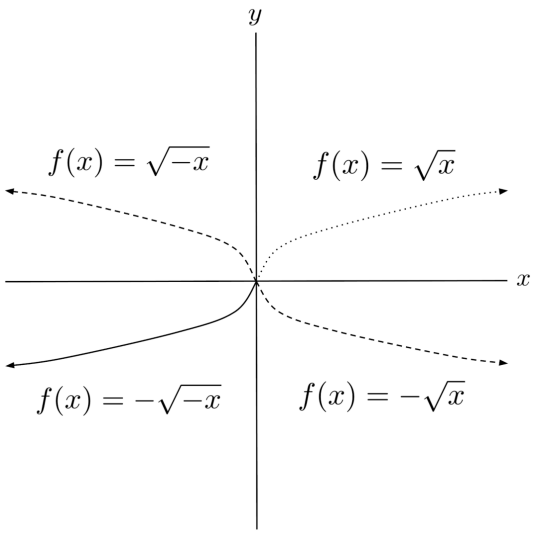
Reflections

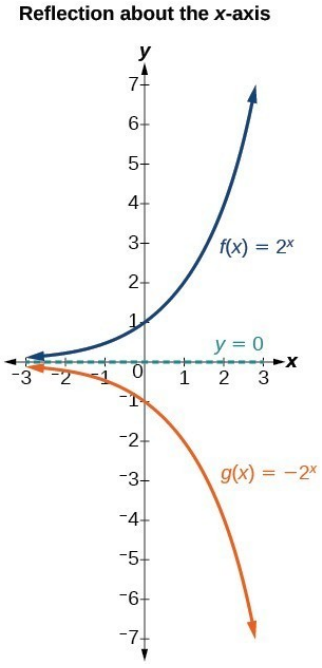
Given a parent function $y = f(x)$:

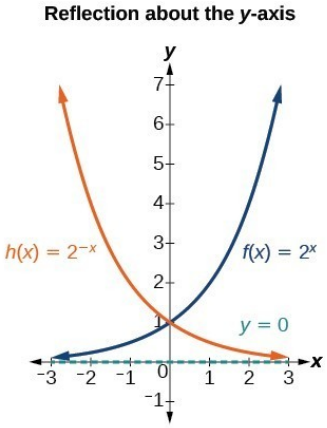
- The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ across the x-axis
- The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ across the y-axis

Examples:

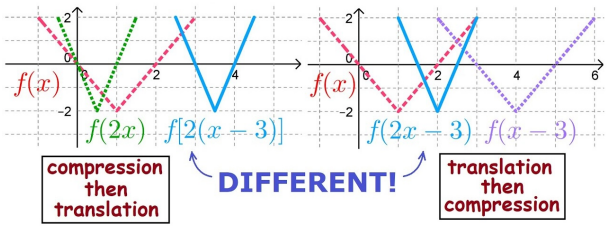








Putting it together



To graph a function by applying more than one transformation, (i.e. to graph $y = af(bx + c) + d$ using $y = f(x)$) use the following order:

(1) Horizontal shifts using c

(2) Horizontal stretching/shrinking and/or reflecting across y-axis using b

(3) Vertical stretching/shrinking and/or reflecting across x-axis using a

(4) Vertical shifts using d

Examples:

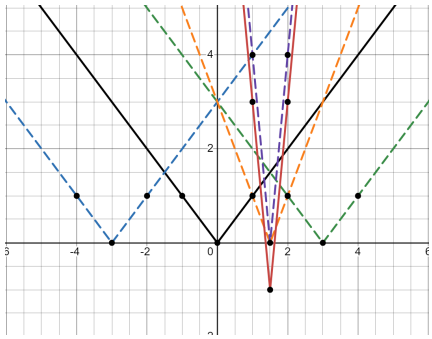
(1) For the graph of $y = \sqrt{-x-2} - 1$, you'd apply the transformations in the following order:

Step 1: Shift the graph of $y = \sqrt{x}$ right 2 units

Step 2: Reflect the graph of across the y-axis

Step 3: Shift the graph down 1 unit

(2) To graph $y = 4|-2x + 3| - 1$ from $y = |x|$



Order:

(1) Black (parent/base graph)

(2) Blue (shifting left 3)

(3) Green (reflecting across the y-axis)

(4) Orange (dividing all x values by 2)

(5) Purple (multiplying all y values by 4)

(6) Red (shifting down 1)

Note: You could swap steps 3 and 4 (both dealing with b)