ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 4.1 Classwork (CW 11) Name: ______________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each group will be given a specific problem that they must complete and present to either Professor MG or to Stefanie before they leave.

(1) Determine if the following are polynomials. If they are, state its degree, leading term, and leading coefficient:

(a) $\dfrac{5x+3}{x}$

(b) $x^2 + 7x^4 + 4x + 9x^3 - 4$

(c) $2x + 3x^{-1} - 5$

(d) $x^2 - x^3 + x^4 - 5$

Solution

(a) This is not a polynomial since there is a division by x

(b) This is a polynomial with degree 4, leading term $7x^4$, and leading coefficient 7

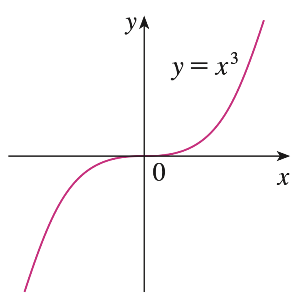
(c) This is not a polynomial since there is a negative exponent on the x

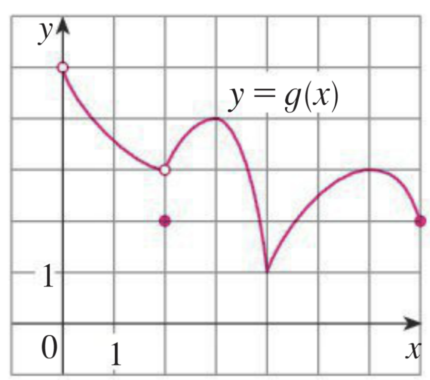
(d) This is a polynomial with degree 4, leading term x^4, and leading coefficient 1

□

(2) Find the relative and absolute extrema of the following graphs. **If there is no solid dot at the end of a graph, assume there is an arrow.**

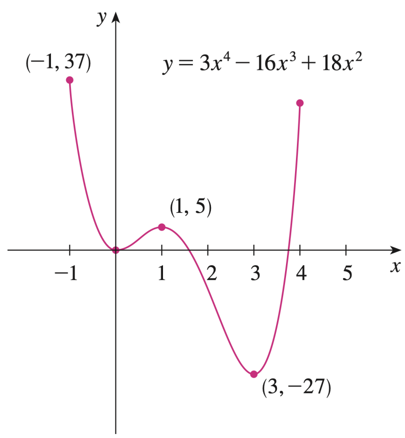
(a)



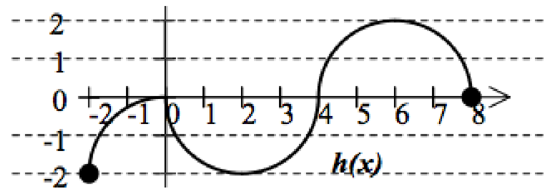


(b)

(c)



(d)



Solution

(a) No relative nor absolute extrema

(b) Relative min and abs min at $(4, 1)$, rel max at $(3, 4)$ and $(6, 3)$, no absolute max

(c) Relative and abs min at $(3, -27)$, rel min at $(0, 0)$, rel max at $(1, 5)$, and abs max $(-1, 37)$

(d) Relative min at $(2, -2)$, abs min at $(-2, -2)$, relative max at $(0, 0)$ relative and abs max at $(6, 2)$

□

(3) Determine if the function is odd, even, or neither.

(a) $f(x) = 3x^2 + 8$

(b) $f(x) = x^5 - 4x$

(c) $f(x) = 2x^2 - x - 1$

(d) $f(x) = \frac{2x}{x^4 + x^2 + 7}$

Solution

(a)

$$\begin{aligned} f(-x) &= 3(-x)^2 + 8 \\ &= 3x^2 + 8 \\ &= f(x) \end{aligned}$$

So the function is $\boxed{\text{even}}$

(b)

$$\begin{aligned} f(-x) &= (-x)^5 - 4(-x) \\ &= -x^5 + 4x \\ &= -f(x) \end{aligned}$$

So the function is $\boxed{\text{odd}}$

(c)

$$\begin{aligned} f(-x) &= 2(-x)^2 - (-x) - 1 \\ &= 2x^2 + x - 1 \end{aligned}$$

This is neither the same as $f(x)$ nor the opposite, so the function is $\boxed{\text{neither}}$

(d)

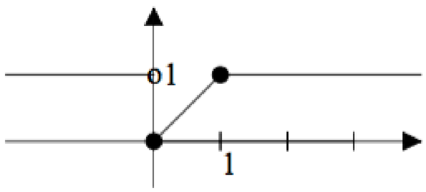
$$\begin{aligned} f(-x) &= \frac{2(-x)}{(-x)^4 + (-x)^2 + 7} \\ &= \frac{-2x}{x^4 + x^2 + 7} \\ &= -f(x) \end{aligned}$$

So the function is $\boxed{\text{odd}}$

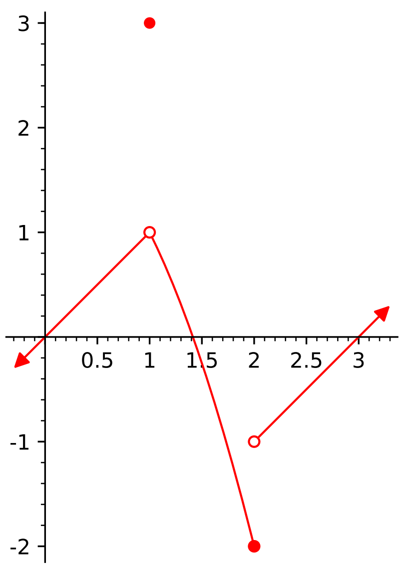
$\square$

(4) Find the domain and range of the following functions. **If there is no solid dot at the end of a graph, assume there is an arrow.**

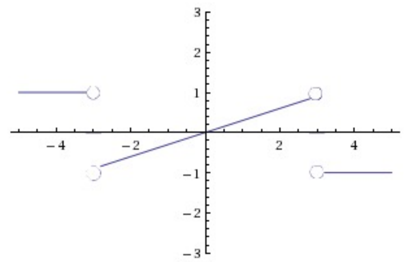
(a)



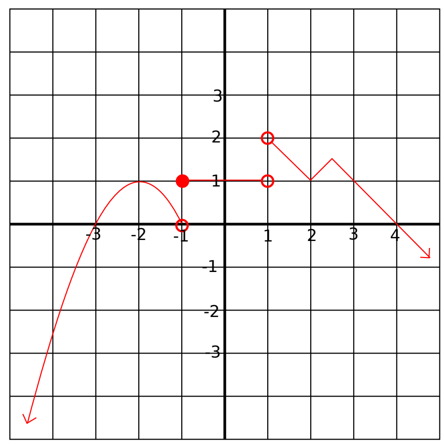
(b)



(c)



(d)



Solution

(a) Domain: $(-\infty, \infty)$, Range: $[0, 1]$

(b) Domain: $(-\infty, \infty)$, Range: $[-2, 1] \cup \{3\}$

(c) Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$, Range: $[-1, 1]$

(d) Domain: $(-\infty, 1) \cup (1, \infty)$, Range: $(-\infty, 2]$

□

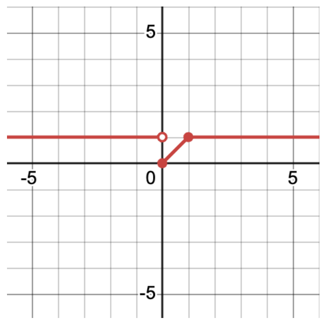
(5) Graph the following piecewise functions by hand **and** find $f(-1)$, $f(0)$, and $f(1)$:

(a) $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x \end{cases}$

(b) $f(x) = \begin{cases} x - 1 & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$

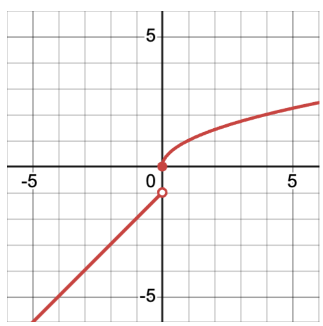
Solution

(a)



$$f(-1) = 1$$
$$f(0) = 0$$
$$f(1) = 1$$

(b)



$$\begin{aligned} f(-1) &= 0 - 1 \\ &= -1 \\ f(0) &= \sqrt{0} \\ &= 0 \\ f(1) &= \sqrt{1} \\ &= 1 \end{aligned}$$

□