ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 4.4 Classwork (CW 14) Name: ____________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each person will be given two specific problems that they must complete and present to either Professor MG or to Stefanie before they leave.

(1) List all the possible rational roots of the following polynomials:

(a) $x^3 + 5x^2 + 8x + 4$
(b) $x^3 - x^2 - 2x + 2$
(c) $3x^3 - 5x^2 + 7x + 3$
(d) $2x^3 - 10x^2 + 15x - 9$

Solution

(a) Using the rational root test, we have options $p = \pm 1, \pm 2, \pm 4$ and $q = \pm 1$. That means that our options for $\frac{p}{q}$ are $\boxed{\pm 1, \pm 2, \pm 4}$

(b) Using the rational root test, we have options $p = \pm 1, \pm 2$ and $q = \pm 1$. That means that our options for $\frac{p}{q}$ are $\boxed{\pm 1, \pm 2}$

(c) Using the rational root test, we have options $p = \pm 1, \pm 3$ and $q = \pm 1, \pm 3$. That means that our options for $\frac{p}{q}$ are $\boxed{\pm 1, \pm 3, \pm \frac{1}{3}}$

(d) Using the rational root test, we have options $p = \pm 1, \pm 3, \pm 9$ and $q = \pm 1, \pm 2$. That means that our options for $\frac{p}{q}$ are $\boxed{\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}}$

$\square$

(2) Find all the roots of the polynomials:

(a) $x^3 + 5x^2 + 8x + 4$ (Given that -1 is a root)

(b) $x^3 - 4x^2 - 9x + 36$ (Given -3 is a root)

(c) $3x^3 - 5x^2 - 16x + 12$ (Given -2 is a root)

(d) $x^3 + x^2 - 7x + 5$ (Given 1 is a root)

Solution

(a) Doing polynomial division with the given root we have:

$$\begin{array}{r|l} & x^2 + 4x + 4 \\ \hline x+1 & x^3 + 5x^2 + 8x + 4 \\ & -x^3 - x^2 \\ \hline & 4x^2 + 8x \\ & -4x^2 - 4x \\ \hline & 4x + 4 \\ & -4x - 4 \\ \hline & 0 \end{array}$$

$$x^3 + 5x^2 + 8x + 4 = (x+1)(x^2 + 4x + 4) = (x+1)(x+2)(x+2)$$

So the roots are $\boxed{x = -1 \text{ and } x = -2}$

(b) Doing polynomial division with the given root we have:

$$\begin{array}{r|l} & x^2 - 7x + 12 \\ \hline x+3 & x^3 - 4x^2 - 9x + 36 \\ & -x^3 - 3x^2 \\ \hline & -7x^2 - 9x \\ & 7x^2 + 21x \\ \hline & 12x + 36 \\ & -12x - 36 \\ \hline & 0 \end{array}$$

$$x^3 - 4x^2 - 9x + 36 = (x+3)(x^2 - 7x + 12) = (x+3)(x-4)(x-3)$$

So the roots are $\boxed{x = -3,\ x = 4, \text{ and } x = 3}$

(c) Doing polynomial division with the given root we have:

$$\begin{array}{r|l} & 3x^2 - 11x \quad + 6 \\ \hline x+2 & 3x^3 \quad - 5x^2 - 16x + 12 \\ & \underline{- 3x^3 \quad - 6x^2} \\ & \qquad - 11x^2 - 16x \\ & \qquad \underline{11x^2 + 22x} \\ & \qquad\qquad 6x + 12 \\ & \qquad\qquad \underline{- 6x - 12} \\ & \qquad\qquad\qquad 0 \end{array}$$

$$3x^3 - 5x^2 - 16x + 12 = (x+2)(3x^2 - 11x + 6) = (x+2)(3x-2)(x-3)$$

So the roots are $\boxed{x = -2,\ x = \frac{2}{3},\ \text{and } x = 3}$

(d) Doing polynomial division with the given root we have:

$$\begin{array}{r|l} & x^2 + 2x - 5 \\ \hline x-1 & x^3 \quad + x^2 - 7x + 5 \\ & \underline{- x^3 \quad + x^2} \\ & \qquad 2x^2 - 7x \\ & \qquad \underline{- 2x^2 + 2x} \\ & \qquad\qquad - 5x + 5 \\ & \qquad\qquad \underline{5x - 5} \\ & \qquad\qquad\qquad 0 \end{array}$$

$$x^3 + x^2 - 7x + 5 = (x+1)(x^2 + 4x + 4) = (x+1)(x+2)(x+2)$$

So the roots are $\boxed{x = -1 \text{ and } x = -2}$

$\square$

(3) List all the roots along with their multiplicities. Then, try to sketch a graph of what you think the function will look like.

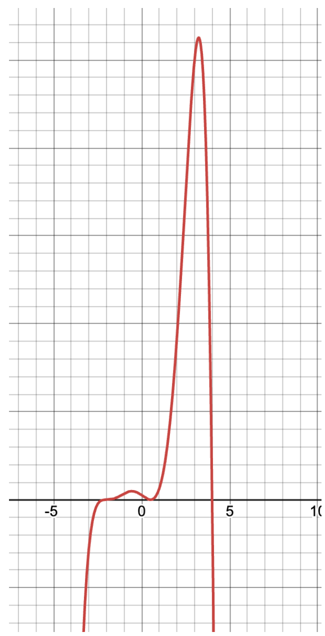
(a) $y = (x+2)^3(4-x)(2x-1)^2$

(b) $f(x) = (x+2)(x-1)^2(x-4)^3$

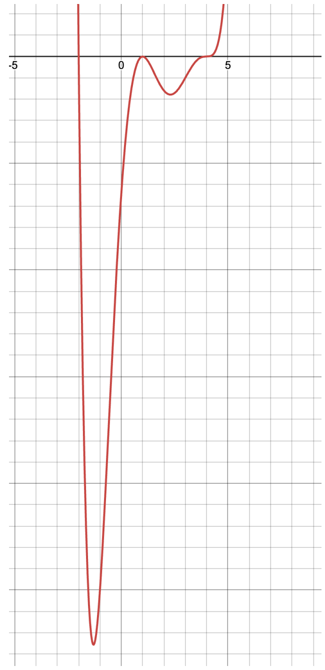
(c) $f(x) = -\frac{1}{3}(x+2)^2(x-3)^3$

Solution

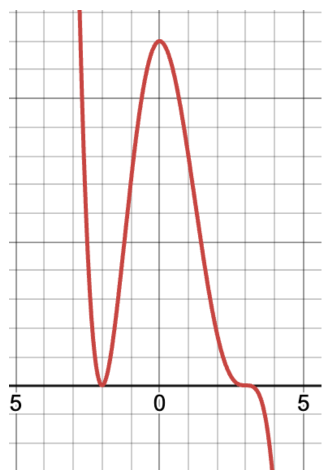
(a) $x = -2$ with multiplicty 3, $x = \frac{1}{2}$ with multiplicity 2, $x = 1$ with multiplicity 1



(b) $x = -2$ with muliplicity 1; $x = 1$ with multiplicity 2; $x = 4$ with multiplicity 3



(c) $x = -2$ with multiplicity 2; $x = 3$ with multiplicity 3



$\square$

(4) Solve the following polynomial equations:

(a) $x^3 + 3x^2 - x - 3 = 0$

(b) $x^3 - 2x^2 = 3x - 6$

(c) $2x^4 - 27x^2 = -3x^3$

Solution

(a)

$$\begin{aligned} x^3 + 3x^2 - x - 3 = 0 &\Leftrightarrow x^2(x+3) - (x+3) = 0 \\ &\Leftrightarrow (x^2-1)(x+3) = 0 \\ &\Leftrightarrow (x-1)(x+1)(x+3) = 0 \\ &\Leftrightarrow \boxed{x = 1, -1, -3} \end{aligned}$$

(b)

$$\begin{aligned} x^3 - 2x^2 = 3x - 6 &\Leftrightarrow x^3 - 2x^2 - 3x + 6 = 0 \\ &\Leftrightarrow x^2(x-2) - 3(x-2) = 0 \\ &\Leftrightarrow (x^2-3)(x-2) = 0 \\ &\Leftrightarrow x^2 = 3 \text{ or } x = 2 \\ &\Leftrightarrow \boxed{x = \pm\sqrt{3}, 2} \end{aligned}$$

(c)

$$\begin{aligned} 2x^4 - 27x^2 = -3x^3 &\Leftrightarrow 2x^4 + 3x^3 - 27x^2 = 0 \\ &\Leftrightarrow x^2(2x^2 + 3x - 27) = 0 \\ &\Leftrightarrow x^2(2x+9)(x-3) = 0 \\ &\Leftrightarrow \boxed{x = 0, -\frac{9}{2}, 3} \end{aligned}$$

□

Key:

(1)
- (a) $\pm1, \pm2, \pm4$
- (b) $\pm1, \pm2$
- (c) $\pm1, \pm3, \pm\frac{1}{3}$
- (d) $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2} \pm \frac{9}{2}$

(2)
- (a) $-1, -2$
- (b) $-3, 3, 4$
- (c) $-2, \frac{2}{3}, 3$
- (d) $1, -1 \pm \sqrt{6}$

(3) Use graphing utility to check your answers
- (a) -2, mult: 3; $\frac{1}{2}$, mult: 2; 1, mult: 1
- (b) -2, mult: 1; 1, mult: 2; 4, mult:3
- (c) -2, mult 2; 3, mult: 3

(4)
- (a) $x = -3, -1, 1$
- (b) $x = 2, \sqrt{3}, -\sqrt{3}$
- (c) $x = 0, 3, -\frac{9}{2}$