ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 4.5 Classwork (CW 15)　　　　Name: ________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each person will be given two specific problems that they must complete and present to either Professor MG or to Stefanie before they leave.

(1) Factor completely: $x^3 + 10x^2 + 169x$

Solution

$$x^3 + 10x^2 + 169x = x(x^2 + 10x + 169)$$

Using the quadratic formula, we get:

$$\begin{aligned} x &= \frac{-10 \pm \sqrt{10^2 - 4(1)(169)}}{2(1)} \\ &= \frac{-10 \pm \sqrt{100 - 676}}{2} \\ &= \frac{-10 \pm \sqrt{-576}}{2} \\ &= \frac{-10 \pm i\sqrt{576}}{2} \\ &= \frac{-10 \pm 24i}{2} \\ &= -5 \pm 12i \end{aligned}$$

So the polynomial factors into $\boxed{x(x - (-5 + 12i))(x - (-5 - 12i))}$

□

(2) Factor completely: $x^3 - 7x^2 + 31x - 25$

Solution Using the rational root test, we know that the possible rational roots are $\pm 1, \pm 5, \pm 25$. Trying $x = 1$, we get

$$\begin{array}{r|l} & x^2 \quad - 6x + 25 \\ \hline x - 1 & x^3 - 7x^2 + 31x - 25 \\ & -x^3 + x^2 \\ \hline & -6x^2 + 31x \\ & 6x^2 - 6x \\ \hline & 25x - 25 \\ & -25x + 25 \\ \hline & 0 \end{array}$$

Using the quadratic formula on the result gives

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)} \\ &= \frac{6 \pm \sqrt{36 - 100}}{2} \\ &= \frac{6 \pm \sqrt{-64}}{2} \\ &= \frac{6 \pm i\sqrt{64}}{2} \\ &= \frac{6 \pm 8i}{2} \\ &= 3 \pm 4i \end{aligned}$$

Therefore the polynomial factors into $\boxed{(x-1)(x-(3+4i))(x-(3-4i))}$

□

(3) Factor completely: $4x^4 + 35x^2 - 9$

Solution Let $y = x^2$, then we can write the polynomial as $4y^2 + 35y - 9$.

$$4y^2 + 35y - 9 = (4y - 1)(y + 9)$$

This means that

$$4x^4 + 35x^2 - 9 = (4x^2 - 1)(x^2 + 9) = (2x - 1)(2x + 1)(x^2 + 9)$$

Using the quadratic formula on the last factor gives us:

$$\begin{aligned} x &= \frac{-0 \pm \sqrt{0^2 - 4(1)(9)}}{2(1)} \\ &= \frac{\pm\sqrt{-36}}{2} \\ &= \frac{\pm i\sqrt{36}}{2} \\ &= \frac{\pm 6i}{2} \\ &= \pm 3i \end{aligned}$$

So our final factorization is $\boxed{(2x - 1)(2x + 1)(x - 3i)(x + 3i)}$

□

(4) Factor completely: $3x^3 + 6x^2 + 6x + 12$

Solution

$$\begin{aligned} 3x^3 + 6x^2 + 6x + 12 &= 3x^2(x+2) + 6(x+2) \\ &= (3x^2 + 6)(x+2) \\ &= 3(x^2+2)(x+2) \end{aligned}$$

Using the quadratic formula on $x^2 + 2$:

$$\begin{aligned} x &= \frac{-0 \pm \sqrt{0^2 - 4(1)(2)}}{2(1)} \\ &= \frac{\pm\sqrt{-8}}{2} \\ &= \frac{\pm i\sqrt{8}}{2} \\ &= \frac{\pm i\sqrt{4 \cdot 2}}{2} \\ &= \frac{\pm 2i\sqrt{2}}{2} \\ &= \pm i\sqrt{2} \end{aligned}$$

So the full factorization is $\boxed{3(x+2)(x-i\sqrt{2})(x+i\sqrt{2})}$

□

(5) Factor completely: $x^5 + 3x^3 - 4x$ given that $x = 1$ is a root.

Solution We can already factor out an x:

$$x^5 + 3x^3 - 4x = x(x^4 + 3x^2 - 4)$$

Since $x = 1$ is a root, we can do polynomial division

$$
\begin{array}{r|l}
 & x^3 + x^2 + 4x + 4 \\
\hline
x - 1 & x^4 \quad + 3x^2 \quad - 4 \\
 & -x^4 + x^3 \\
\hline
 & x^3 + 3x^2 \\
 & -x^3 + x^2 \\
\hline
 & 4x^2 \\
 & -4x^2 + 4x \\
\hline
 & 4x - 4 \\
 & -4x + 4 \\
\hline
 & 0
\end{array}
$$

$$
\begin{aligned}
x^5 + 3x^3 - 4x &= x(x^4 - 3x^2 - 4) \\
&= x(x-1)(x^3 + x^2 + 4x + 4) \\
&= x(x-1)[x^2(x+1) + 4(x+1)) \\
&= x(x-1)(x+1)(x^2+4)
\end{aligned}
$$

Using the quadratic formula on $x^2 + 4$ will give $x = \pm 2i$, so the full factorization is $\boxed{x(x-1)(x+1)(x+2i)(x-2i)}$

□

(6) Factor completely: $x^4 - 2x^3 + 6x^2 - 8x + 8$ given that $2i$ is a root

Solution If $2i$ is a root, then its conjugate $-2i$ is also a root. This means that one of the factors can be written as

$$(x - 2i)(x + 2i) = x^2 + 4$$

Doing polynomial long division with this we have

$$\begin{array}{r l}
 & \; x^2 - 2x + 2 \\
\cline{2-2}
x^2 + 4 & \big)\; x^4 - 2x^3 + 6x^2 - 8x + 8 \\
 & \;\; - x^4 \qquad - 4x^2 \\
\cline{2-2}
 & \qquad - 2x^3 + 2x^2 - 8x \\
 & \qquad\;\; 2x^3 \qquad + 8x \\
\cline{2-2}
 & \qquad\qquad 2x^2 + 0x + 8 \\
 & \qquad\qquad - 2x^2 \qquad - 8 \\
\cline{2-2}
 & \qquad\qquad\qquad 0x + 0
\end{array}$$

Using the quadratic formula we have

$$\begin{aligned}
x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} \\
&= \frac{2 \pm \sqrt{4 - 8}}{2} \\
&= \frac{2 \pm \sqrt{-4}}{2} \\
&= \frac{2 \pm i\sqrt{4}}{2} \\
&= \frac{2 \pm 2i}{2} \\
&= 1 \pm i
\end{aligned}$$

This gives us $\boxed{(x - 2i)(x + 2i)(x - (1 + i))(x - (1 - i))}$

□

(7) Solve $x^3 - 8 = 0$ (including complex solutions)

Solution We already know that a solution to this is $x = 2$ if we try to solve it normally. So we can use polynomial long division

$$\begin{array}{r|l} & x^2 + 2x + 4 \\ x - 2 & x^3 \qquad\qquad - 8 \\ & -x^3 + 2x^2 \\ \hline & 2x^2 \\ & -2x^2 + 4x \\ \hline & 4x - 8 \\ & -4x + 8 \\ \hline & 0 \end{array}$$

Using the quadratic formula we have

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{4 - 16}}{2} \\ &= \frac{-2 \pm \sqrt{-12}}{2} \\ &= \frac{-2 \pm i\sqrt{12}}{2} \\ &= \frac{-2 \pm 2i\sqrt{3}}{2} \\ &= -1 \pm i\sqrt{3} \end{aligned}$$

So our final answer is $\boxed{(x-2)(x-(-1+i\sqrt{3}))(x-(-1-i\sqrt{3}))}$

□

(8) Solve $x^3 + 25x = 4x^2 + 100$ (including complex solutions)

Solution

$$\begin{aligned} x^3 + 25x = 4x^2 + 100 &\Leftrightarrow x^3 - 4x^2 + 25x - 100 = 0 \\ &\Leftrightarrow x^2(x-4) + 25(x-4) = 0 \\ &\Leftrightarrow (x^2+25)(x-4) = 0 \\ &\Leftrightarrow x^2 + 25 = 0 \text{ or } x - 4 = 0 \\ &\Leftrightarrow x^2 = -25 \text{ or } x = 4 \\ &\Leftrightarrow x = \pm\sqrt{-25} \text{ or } x = 4 \\ &\Leftrightarrow x = \pm i\sqrt{25} \text{ or } x = 4 \\ &\Leftrightarrow x = \pm 5i, 4 \end{aligned}$$

□