ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 4.6 Classwork (CW 16) Name: ____________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each person will be given two specific problems that they must complete and present to either Professor MG or to Stefanie before they leave.

(1) Graph the following function by hand: $y = \frac{2x-6}{2x^3-8x^2}$

Solution
x-intercept(s):

$$\begin{aligned} y = 0 &\Leftrightarrow \frac{2x-6}{2x^3-8x^2} = 0 \\ &\Leftrightarrow 2x - 6 = 0 \\ &\Leftrightarrow 2x = 6 \\ &\Leftrightarrow x = 3 \end{aligned}$$

y-intercept:

$$\begin{aligned} f(0) &= \frac{2(0)-6}{2(0)^3-8(0)^2} \\ &= \frac{-6}{0} \\ &= \text{undefined} \end{aligned}$$

Vertical asymptote(s):

$$\begin{aligned} 2x^3 - 8x^2 = 0 &\Leftrightarrow 2x^2(x-4) = 0 \\ &\Leftrightarrow x^2 = 0 \text{ or } x - 4 = 0 \\ &\Leftrightarrow x = 0 \text{ or } x = 4 \end{aligned}$$

Horizontal/Slant asymptote(s): The degree of the numerator is less than the degree of the denominator, so $y = 0$ is the horizontal asymptote.

On a number line, we would plot the x values of 0, 3, and 4. Testing some values (for example), we could have:

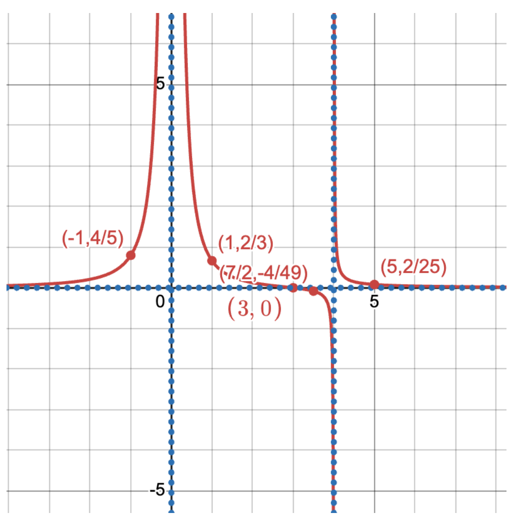
$$f(-1) = \frac{2(-1)-6}{2(-1)^3-8(-1)^2} = \frac{-2-6}{-2-8} = \frac{-8}{-10} = \frac{4}{5}$$

$$f(1) = \frac{2(1)-6}{2(1)^3-8(1)^2} = \frac{2-6}{2-8} = \frac{-4}{-6} = \frac{2}{3}$$

$$f\left(\frac{7}{2}\right) = \frac{2\left(\frac{7}{2}\right)-6}{2\left(\frac{7}{2}\right)^3-8\left(\frac{7}{2}\right)^2} = \frac{7-6}{2\left(\frac{343}{8}\right)-8\left(\frac{49}{4}\right)} = \frac{1}{\frac{343}{4}-\frac{392}{4}} = \frac{1}{-\frac{49}{4}} = -\frac{4}{49}$$

$$f(5) = \frac{2(5)-6}{2(5)^3-8(5)^2} = \frac{10-6}{2(125)-8(25)} = \frac{4}{250-200} = \frac{4}{50} = \frac{2}{25}$$

Using these points, we would get the following sketch:



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(2) Graph the following function by hand: $f(x) = \frac{2x^3-8x^2}{2x-6}$

Solution

x-intercept(s)

$$y = 0 \Leftrightarrow \frac{2x^3 - 8x^2}{2x - 6} = 0$$
$$\Leftrightarrow 2x^3 - 8x^2 = 0$$
$$\Leftrightarrow 2x^2(x - 4) = 0$$
$$\Leftrightarrow x = 0 \text{ or } x = 4$$

y-intercept

$$f(0) = \frac{2(0)^3 - 8(0)^2}{2(0) - 6}$$
$$= \frac{0 - 0}{0 - 6}$$
$$= \frac{0}{-6}$$
$$= 0$$

Vertical asymptote(s):

$$2x - 6 = 0 \Leftrightarrow 2x = 6$$
$$\Leftrightarrow x = 3$$

Horizontal/Slant asymptote: The degree of the numerator is larger (by 2) than the degree of the denominator, so there is no horizontal asymptote and also no slant asymptote.

On a number line we would plot $x = 0, 3$, and 4. Testing some values, we could have:

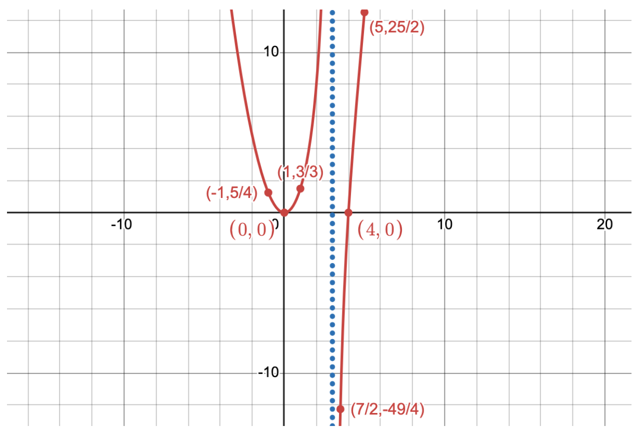
$$f(-1) = \frac{2(-1)^3 - 8(-1)^2}{2(-1) - 6} = \frac{-2 - 8}{-2 - 6} = \frac{-10}{-8} = \frac{5}{4}$$

$$f(1) = \frac{2(1)^3 - 8(1)^2}{2(1) - 6} = \frac{2 - 8}{2 - 6} = \frac{-6}{-4} = \frac{3}{2}$$

$$
\begin{aligned}
f\left(\frac{7}{2}\right) &= \frac{2\left(\frac{7}{2}\right)^3 - 8\left(\frac{7}{2}\right)^2}{2\left(\frac{7}{2}\right) - 6} \\
&= \frac{2\left(\frac{343}{8}\right) - 8\left(\frac{49}{4}\right)}{7 - 6} \\
&= \frac{\frac{343}{4} - \frac{392}{4}}{1} \\
&= \frac{49}{4} \\
&= -\frac{49}{4}
\end{aligned}
$$

$$
\begin{aligned}
f(5) &= \frac{2(5)^3 - 8(5)^2}{2(5) - 6} \\
&= \frac{2(125) - 8(25)}{10 - 6} \\
&= \frac{250 - 200}{4} \\
&= \frac{50}{4} \\
&= \frac{25}{2}
\end{aligned}
$$

Using these points, we would get the following sketch:



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(3) Graph the following function by hand: $f(x) = \frac{3x^2+1}{x^2-4}$

Solution

<u>x-intercept(s)</u>

$$\begin{aligned} y = 0 &\Leftrightarrow \frac{3x^2+1}{x^2-4} = 0 \\ &\Leftrightarrow 3x^2 + 1 = 0 \\ &\Leftrightarrow 3x^2 = -1 \\ &\Leftrightarrow x^2 = -\frac{1}{3} \\ &\Leftrightarrow x = \pm\sqrt{-\frac{1}{3}} \text{ (Not a real number, so no } x \text{ intercept)} \end{aligned}$$

<u>y-intercept</u>

$$\begin{aligned} f(0) &= \frac{3(0)^2+1}{(0)^2-4} \\ &= \frac{0+1}{0-4} \\ &= -\frac{1}{4} \end{aligned}$$

<u>Vertical asymptote(s):</u>

$$\begin{aligned} x^2 - 4 = 0 &\Leftrightarrow (x-2)(x+2) = 0 \\ &\Leftrightarrow x = 2 \text{ or } x = -2 \end{aligned}$$

<u>Horizontal/Slant asymptote:</u> The degree of the numerator and denominator are equal, so the horizontal asymptote is

$$\begin{aligned} y &= \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

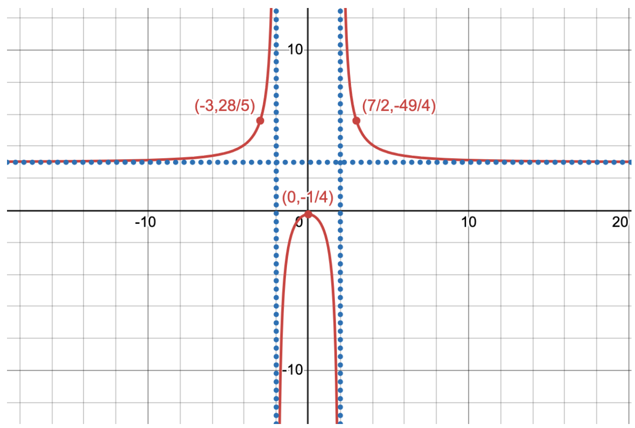
On a number line we would plot $x = -2$ and $x = 2$. Testing some values, we could have:

$$\begin{aligned} f(-3) &= \frac{3(-3)^2+1}{(-3)^2-4} \\ &= \frac{3(9)+1}{9-4} \\ &= \frac{28}{5} \end{aligned}$$

$$\begin{aligned} f(0) &= \frac{3(0)^2+1}{(0)^2-4} \\ &= \frac{0+1}{0-4} \\ &= -\frac{1}{4} \end{aligned}$$

$$f(3) = \frac{3(3)^2 + 1}{(3)^2 - 4}$$
$$= \frac{3(9) + 1}{9 - 4}$$
$$= \frac{28}{5}$$

Using these points, we would get the following sketch:



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(4) Graph the following function by hand: $f(x) = \frac{10x^3}{x^2-1}$

Solution

x-intercept(s)

$$\begin{aligned} y = 0 &\Leftrightarrow \frac{10x^3}{x^2-1} = 0 \\ &\Leftrightarrow 10x^3 = 0 \\ &\Leftrightarrow x^3 = 0 \\ &\Leftrightarrow x = 0 \end{aligned}$$

y-intercept

$$\begin{aligned} f(0) &= \frac{10(0)^3}{(0)^2-1} \\ &= \frac{0}{-1} \\ &= 0 \end{aligned}$$

Vertical asymptote(s):

$$\begin{aligned} x^2 - 1 = 0 &\Leftrightarrow (x-1)(x+1) = 0 \\ &\Leftrightarrow x = 1 \text{ or } x = -1 \end{aligned}$$

Horizontal/Slant asymptote: The degree of the numerator is exactly one more than the degree of the denominator, so there is a slant asymptote. Using polynomial division we have:

$$\begin{array}{r@{}l} & \quad\ \ 10x \\ x^2-1\big) & \ 10x^3 \\ & -10x^3 + 10x \\ \hline & \qquad\ 10x \end{array}$$

So the equation of the slant asymptote is $y = 10x$.

On a number line we would plot $x = -1, 0$, and 1. Testing some values, we could have:

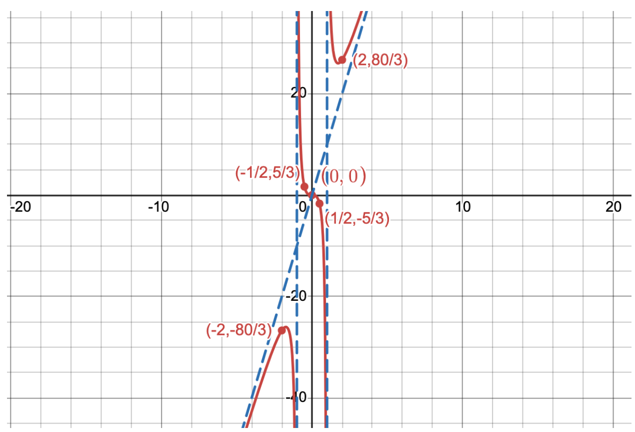
$$\begin{aligned} f(-2) &= \frac{10(-2)^3}{(-2)^2-1} \\ &= \frac{10(-8)}{4-1} \\ &= \frac{-80}{3} \end{aligned}$$

$$\begin{aligned} f(2) &= \frac{10(2)^3}{(-2)^2-1} \\ &= \frac{10(8)}{4-1} \\ &= \frac{80}{3} \end{aligned}$$

$$
\begin{aligned}
f\left(-\frac{1}{2}\right) &= \frac{10\left(\frac{-1}{2}\right)^3}{\left(-\frac{1}{2}\right)^2 - 1} \\
&= \frac{10\left(\frac{-1}{8}\right)}{\frac{1}{4} - 1} \\
&= \frac{\frac{-10}{8}}{\frac{1}{4} - \frac{4}{4}} \\
&= \frac{\frac{-5}{4}}{-\frac{3}{4}} \\
&= -\frac{5}{4} \cdot \frac{-4}{3} \\
&= \frac{5}{3}
\end{aligned}
$$

$$
\begin{aligned}
f\left(\frac{1}{2}\right) &= \frac{10\left(\frac{1}{2}\right)^3}{\left(\frac{1}{2}\right)^2 - 1} \\
&= \frac{10\left(\frac{1}{8}\right)}{\frac{1}{4} - 1} \\
&= \frac{\frac{10}{8}}{\frac{1}{4} - \frac{4}{4}} \\
&= \frac{\frac{5}{4}}{-\frac{3}{4}} \\
&= \frac{5}{4} \cdot \frac{-4}{3} \\
&= -\frac{5}{3}
\end{aligned}
$$

Using these points, we would get the following sketch:



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(5) Graph the following function by hand: $y = \frac{-3x^2+12}{x-1}$

Solution

x-intercept(s)

$$\begin{aligned} y = 0 \Leftrightarrow \frac{-3x^2+12}{x-1} &= 0 \\ \Leftrightarrow -3x^2 + 12 &= 0 \\ \Leftrightarrow -3(x^2-4) &= 0 \\ \Leftrightarrow -3(x-2)(x+2) &= 0 \\ \Leftrightarrow x = 2 \text{ or } x &= -2 \end{aligned}$$

y-intercept

$$\begin{aligned} f(0) &= \frac{-3(0)^2+12}{0-1} \\ &= \frac{0+12}{-1} \\ &= -12 \end{aligned}$$

Vertical asymptote(s):

$$x - 1 = 0 \Leftrightarrow x = 1$$

Horizontal/Slant asymptote: The degree of the numerator is exactly one more than the degree of the denominator, so there is a slant asymptote. Using polynomial division we have:

$$\begin{array}{r|l} & -3x \quad -3 \\ \hline x-1 & -3x^2 \qquad +12 \\ & 3x^2 - 3x \\ \hline & -3x + 12 \\ & 3x \quad -3 \\ \hline & 9 \end{array}$$

So the equation of the slant asymptote is $y = -3x - 3$.

On a number line we would plot $x = -2, 1$ and 2. Testing some values, we could have:

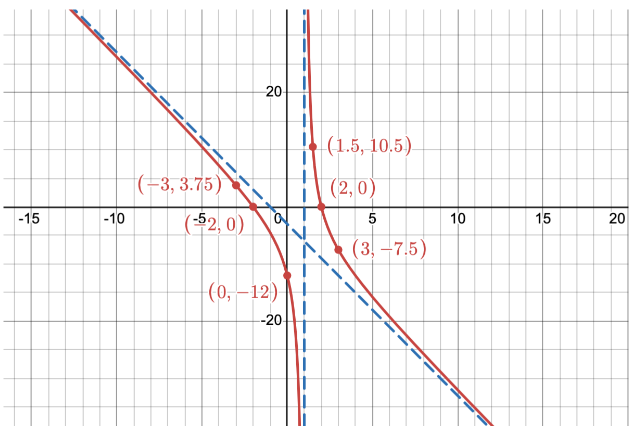
$$\begin{aligned} f(-3) &= \frac{-3(-3)^2+12}{-3-1} \\ &= \frac{-3(9)+12}{-4} \\ &= \frac{-27+12}{-4} \\ &= \frac{-15}{-4} \\ &= \frac{15}{4} \end{aligned}$$

$$\begin{aligned} f(3) &= \frac{-3(3)^2+12}{3-1} \\ &= \frac{-3(9)+12}{2} \\ &= \frac{-27+12}{2} \\ &= \frac{-15}{2} \end{aligned}$$

$$\begin{aligned} f(0) &= \frac{-3(0)^2+12}{0-1} \\ &= \frac{0+12}{-1} \\ &= -12 \end{aligned}$$

$$\begin{aligned} f\left(\frac{3}{2}\right) &= \frac{-3\left(\frac{3}{2}\right)^2+12}{\frac{3}{2}-1} \\ &= \frac{-3\left(\frac{9}{4}\right)+12}{\frac{3}{2}-\frac{2}{2}} \\ &= \frac{\frac{-27}{4}+12}{\frac{1}{2}} \\ &= \frac{\frac{-27}{4}+\frac{48}{4}}{\frac{1}{2}} \\ &= \frac{\frac{21}{4}}{\frac{1}{2}} \\ &= \frac{21}{4}\cdot\frac{2}{1} \\ &= \frac{21}{2} \end{aligned}$$

Using these points, we would get the following sketch:



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(6) Graph the following function by hand: $y = \frac{x^2-4}{x^2-4x}$

Solution

x-intercept(s)

$$\begin{aligned} y = 0 &\Leftrightarrow \frac{x^2-4}{x^2-4x} = 0 \\ &\Leftrightarrow x^2 - 4 = 0 \\ &\Leftrightarrow (x-2)(x+2) = 0 \\ &\Leftrightarrow x = 2 \text{ or } x = -2 \end{aligned}$$

y-intercept

$$\begin{aligned} f(0) &= \frac{(0)^2 - 4}{(0)^2 - 4(0)} \\ &= \frac{-4}{0} \\ &= \text{undefined} \end{aligned}$$

Vertical asymptote(s):

$$\begin{aligned} x^2 - 4x = 0 &\Leftrightarrow x(x-4) = 0 \\ &\Leftrightarrow x = 0 \text{ or } x = 4 \end{aligned}$$

Horizontal/Slant asymptote: The degree of the numerator and denominator are equal, so the horizontal asymptote is

$$\begin{aligned} y &= \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

On a number line we would plot $x = -2, 0, 2, 4$. Testing some values, we could have: Using these points, we would get the following sketch:

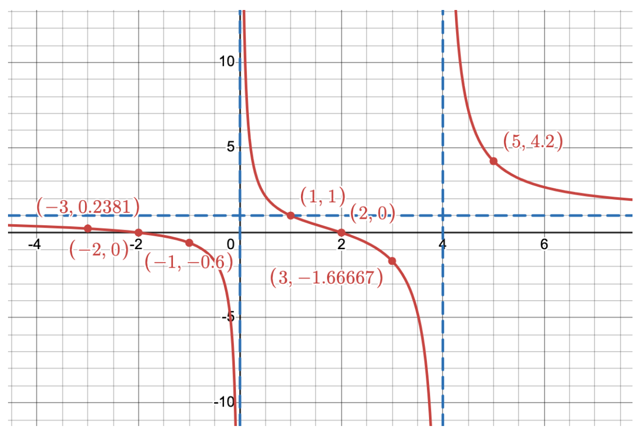
$$\begin{aligned} f(-3) &= \frac{(-3)^2 - 4}{(-3)^2 - 4(-3)} \\ &= \frac{9-4}{9+12} \\ &= \frac{5}{21} \end{aligned}$$

$$\begin{aligned} f(-1) &= \frac{(-1)^2 - 4}{(-1)^2 - 4(-1)} \\ &= \frac{1-4}{1+4} \\ &= \frac{-3}{5} \end{aligned}$$

$$f(1) = \frac{(1)^2 - 4}{(1)^2 - 4(1)}$$
$$= \frac{1 - 4}{1 - 4}$$
$$= 1$$

$$f(3) = \frac{(3)^2 - 4}{(3)^2 - 4(3)}$$
$$= \frac{9 - 4}{9 - 12}$$
$$= \frac{5}{-3}$$

$$f(5) = \frac{(5)^2 - 4}{(5)^2 - 4(5)}$$
$$= \frac{25 - 4}{25 - 20}$$
$$= \frac{21}{5}$$



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(7) Graph the following function by hand: $f(x) = \frac{x-2}{x^2-3x-4}$

Solution

x-intercept(s)

$$\begin{aligned} y = 0 &\Leftrightarrow \frac{x-2}{x^2-3x-4} = 0 \\ &\Leftrightarrow x - 2 = 0 \\ &\Leftrightarrow x = 2 \end{aligned}$$

y-intercept

$$\begin{aligned} f(0) &= \frac{0-2}{(0)^2 - 3(0) - 4} \\ &= \frac{-2}{-4} \\ &= \frac{1}{2} \end{aligned}$$

Vertical asymptote(s):

$$\begin{aligned} x^2 - 3x - 4 = 0 &\Leftrightarrow (x-4)(x+1) = 0 \\ &\Leftrightarrow x = 4 \text{ or } x = -1 \end{aligned}$$

Horizontal/Slant asymptote: The degree of the numerator is less than the degree of the denominator, so $y = 0$ is the horizontal asymptote.

On a number line we would plot $x = -1, 2$ and 4. Testing some values, we could have:

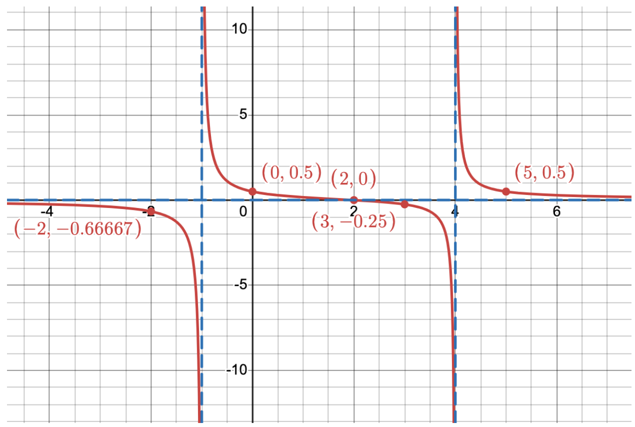
$$\begin{aligned} f(-2) &= \frac{-2-2}{(-2)^2 - 3(-2) - 4} \\ &= \frac{-4}{4+6-4} \\ &= \frac{-4}{6} \\ &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} f(0) &= \frac{0-2}{(0)^2 - 3(0) - 4} \\ &= \frac{-2}{-4} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} f(3) &= \frac{3-2}{(3)^2 - 3(3) - 4} \\ &= \frac{1}{9-9-4} \\ &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} f(5) &= \frac{5-2}{(5)^2 - 3(5) - 4} \\ &= \frac{3}{25-15-4} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

Using these points, we would get the following sketch:



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(8) Graph the following function by hand: $y = \frac{x^4-3x^3+2x^2}{x^2-2x-3}$

Solution

x-intercept(s)

$$\begin{aligned} y = 0 &\Leftrightarrow \frac{x^4-3x^3+2x^2}{x^2-2x-3} = 0 \\ &\Leftrightarrow x^4-3x^3+2x^2 = 0 \\ &\Leftrightarrow x^2(x^2-3x+2) = 0 \\ &\Leftrightarrow x^2(x-2)(x-1) = 0 \\ &\Leftrightarrow x = 0 \text{ or } x = 2 \text{ or } x = 1 \end{aligned}$$

y-intercept

$$\begin{aligned} f(0) &= \frac{(0)^4-3(0)^3+2(0)^2}{(0)^2-2(0)-3} \\ &= \frac{0}{-3} \\ &= 0 \end{aligned}$$

Vertical asymptote(s):

$$\begin{aligned} x^2-2x-3 = 0 &\Leftrightarrow (x-3)(x+1) = 0 \\ &\Leftrightarrow x = 3 \text{ or } x = -1 \end{aligned}$$

Horizontal/Slant asymptote: The degree of the numerator is larger (by 2) than the degree of the denominator, so there is no horizontal asymptote and also no slant asymptote.

On a number line we would plot $x = -1, 0, 1, 2$, and 3. Testing some values, we could have:

$$\begin{aligned} f(-2) &= \frac{(-2)^4-3(-2)^3+2(-2)^2}{(-2)^2-2(-2)-3} \\ &= \frac{16-3(-8)+2(4)}{4+4-3} \\ &= \frac{16+24+8}{5} \\ &= \frac{48}{5} \end{aligned}$$

$$\begin{aligned} f(4) &= \frac{(4)^4-3(4)^3+2(4)^2}{(4)^2-2(4)-3} \\ &= \frac{256-3(64)+2(16)}{16-8-3} \\ &= \frac{256-192+32}{5} \\ &= \frac{96}{5} \end{aligned}$$

$$f\left(-\frac{1}{2}\right) = \frac{\left(-\frac{1}{2}\right)^4 - 3\left(-\frac{1}{2}\right)^3 + 2\left(-\frac{1}{2}\right)^2}{\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 3}$$

$$= \frac{\frac{1}{16} - 3\left(-\frac{1}{8}\right) + 2\left(\frac{1}{4}\right)}{\frac{1}{4} + 1 - 3}$$

$$= \frac{\frac{1}{16} + \frac{3}{8} + \frac{1}{2}}{\frac{1}{4} - 2}$$

$$= \frac{\frac{1}{16} + \frac{6}{16} + \frac{8}{16}}{\frac{1}{4} - \frac{8}{4}}$$

$$= \frac{\frac{15}{16}}{\frac{-7}{4}}$$

$$= \frac{15}{16} \cdot -\frac{4}{7}$$

$$= -\frac{15}{28}$$

$$f\left(\frac{5}{2}\right) = \frac{\left(\frac{5}{2}\right)^4 - 3\left(\frac{5}{2}\right)^3 + 2\left(\frac{5}{2}\right)^2}{\left(\frac{5}{2}\right)^2 - 2\left(\frac{5}{2}\right) - 3}$$

$$= \frac{\frac{625}{16} - 3\left(\frac{125}{8}\right) + 2\left(\frac{25}{4}\right)}{\frac{25}{4} - 5 - 3}$$

$$= \frac{\frac{625}{16} - \frac{375}{8} + \frac{50}{4}}{\frac{25}{4} - 8}$$

$$= \frac{\frac{625}{16} - \frac{750}{16} + \frac{200}{16}}{\frac{25}{4} - \frac{32}{4}}$$

$$= \frac{\frac{75}{16}}{\frac{-7}{4}}$$

$$= \frac{75}{16} \cdot -\frac{4}{7}$$

$$= -\frac{75}{28}$$

$$f\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^4 - 3\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) - 3}$$

$$= \frac{\frac{1}{16} - 3\left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right)}{\frac{1}{4} - 2\left(\frac{1}{2}\right) - 3}$$

$$= \frac{\frac{1}{16} - \frac{3}{8} + \frac{1}{2}}{\frac{1}{4} - 1 - 3}$$

$$= \frac{\frac{1}{16} - \frac{6}{16} + \frac{8}{16}}{\frac{1}{4} - 4}$$

$$= \frac{\frac{3}{16}}{\frac{1}{4} - \frac{16}{4}}$$

$$= \frac{\frac{3}{16}}{-\frac{15}{4}}$$

$$= \frac{3}{16} \cdot -\frac{4}{15}$$

$$= -\frac{1}{20}$$

$$f\left(\frac{3}{2}\right) = \frac{\left(\frac{3}{2}\right)^4 - 3\left(\frac{3}{2}\right)^3 + 2\left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right) - 3}$$

$$= \frac{\frac{81}{16} - 3\left(\frac{27}{8}\right) + 2\left(\frac{9}{4}\right)}{\frac{9}{4} - 3 - 3}$$

$$= \frac{\frac{81}{16} - \frac{81}{8} + \frac{18}{4}}{\frac{9}{4} - 6}$$

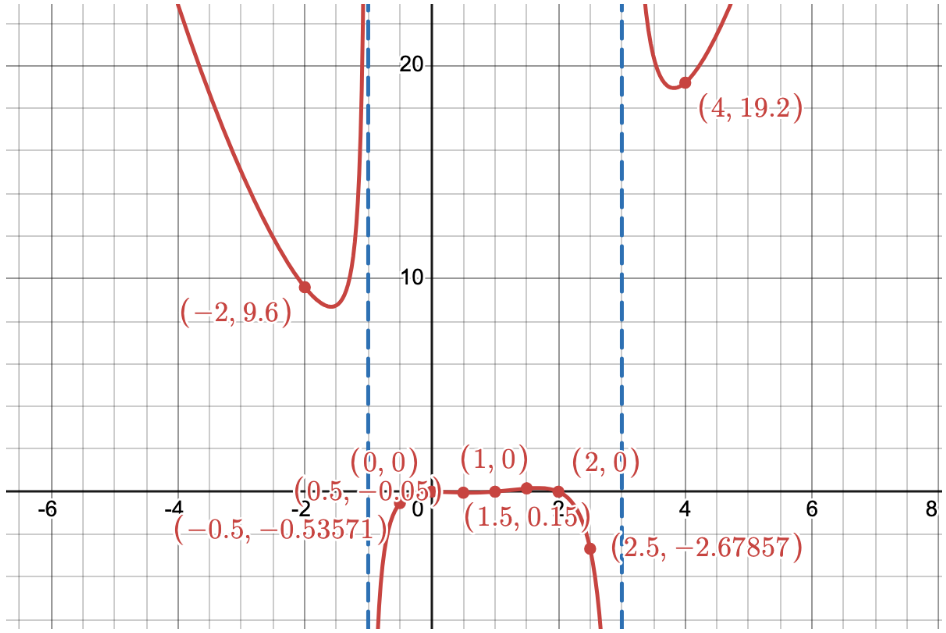
$$= \frac{\frac{81}{16} - \frac{162}{16} + \frac{72}{16}}{\frac{9}{4} - \frac{24}{4}}$$

$$= \frac{\frac{-9}{16}}{-\frac{15}{4}}$$

$$= -\frac{9}{16} \cdot -\frac{4}{15}$$

$$= \frac{9}{60}$$

Using these points, we would get the following sketch:



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