ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 4.7 Classwork (CW 17) Name: ______________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each person will be given two specific problems that they must complete and present to either Professor MG or to Stefanie before they leave.

(1) Solve $\frac{3}{x+5} + \frac{4}{x} = 2$

Solution Note: You cannot divide by 0, so $x \neq -5, 0$

$$\begin{aligned}
\frac{3}{x+5} + \frac{4}{x} = 2 &\Leftrightarrow x(x+5)\left(\frac{3}{x+5} + \frac{4}{x}\right) = 2x(x+5) \\
&\Leftrightarrow \frac{3x\cancel{(x+5)}}{\cancel{(x+5)}} + \frac{4\cancel{x}(x+5)}{\cancel{x}} = 2x(x+5) \\
&\Leftrightarrow 3x + 4(x+5) = 2x(x+5) \\
&\Leftrightarrow 3x + 4x + 20 = 2x^2 + 10x \\
&\Leftrightarrow 2x^2 + 3x - 20 = 0 \\
&\Leftrightarrow (2x-5)(x+4) = 0 \\
&\Leftrightarrow \boxed{x = \frac{5}{2}, -4}
\end{aligned}$$

□

(2) Solve $1 - x - \frac{2}{6x+1} = 0$

Solution Note: You cannot divide by 0 so $x \neq -\frac{1}{6}$

$$\begin{aligned}
1 - x - \frac{2}{6x+1} = 0 &\Leftrightarrow (6x+1)\left(1 - x - \frac{2}{6x+1}\right) = 0(6x+1) \\
&\Leftrightarrow (6x+1) - x(6x+1) - \frac{2\cancel{(6x+1)}}{\cancel{6x+1}} = 0 \\
&\Leftrightarrow 6x + 1 - 6x^2 - x - 2 = 0 \\
&\Leftrightarrow -6x^2 + 5x - 1 = 0 \\
&\Leftrightarrow 6x^2 - 5x + 1 = 0 \\
&\Leftrightarrow (3x-1)(2x-1) = 0 \\
&\Leftrightarrow \boxed{x = \frac{1}{3}, \frac{1}{2}}
\end{aligned}$$

□

(3) Solve $\frac{3x^2-6x-3}{(x+1)(x-2)(x-3)} + \frac{5-2x}{x^2-5x+6} = 0$

Solution

$$\frac{3x^2-6x-3}{(x+1)(x-2)(x-3)} + \frac{5-2x}{x^2-5x+6} = 0 \Leftrightarrow \frac{3x^2-6x-3}{(x+1)(x-2)(x-3)} + \frac{5-2x}{(x-3)(x-2)} = 0$$

Note: You cannot divide by 0, so $x \neq -1, 2, 3$. We'll start by multiplying by the LCD

$$\begin{aligned}(x+1)(x-2)(x-3)\left(\frac{3x^2-6x-3}{(x+1)(x-2)(x-3)} + \frac{5-2x}{(x-3)(x-2)} = 0\right) &\Leftrightarrow 3x^2-6x-3+(x+1)(5-2x) = 0\\ &\Leftrightarrow 3x^2-6x-3+5x-2x^2+5-2x = 0\\ &\Leftrightarrow x^2-3x+2 = 0\\ &\Leftrightarrow (x-2)(x-1) = 0\\ &\Leftrightarrow x = 2, 1\end{aligned}$$

Remember that $x \neq 2$, so the only solution is $\boxed{x = 1}$

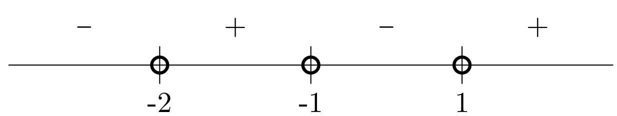
□

(4) Solve $x^3 + 2x^2 - x - 2 > 0$

Solution

$$\begin{aligned}x^3 + 2x^2 - x - 2 > 0 &\Leftrightarrow x^2(x+2) - (x+2) > 0\\ &\Leftrightarrow (x^2-1)(x+2) > 0\\ &\Leftrightarrow (x-1)(x+1)(x+2) > 0\end{aligned}$$

The numbers to put on our number line are therefore $x = 1, -1, -2$. Testing the intervals we have:



We want the parts that are > 0 (positive), so $\boxed{(-2,-1) \cup (1, \infty)}$

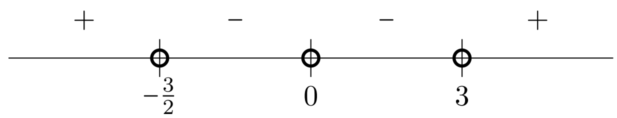
□

(5) Solve $2x^4 > 3x^3 + 9x^2$

Solution

$$\begin{aligned} 2x^4 > 3x^3 + 9x^2 &\Leftrightarrow 2x^4 - 3x^3 - 9x^2 > 0 \\ &\Leftrightarrow x^2(2x^2 - 3x - 9) > 0 \\ &\Leftrightarrow x^2(2x+3)(x-3) > 0 \end{aligned}$$

The numbers to put on our number line are therefore $x = 0, -\frac{3}{2}, 3$. Testing the intervals we have:



We want the parts that are > 0 (positive), so $\boxed{\left(-\infty, -\frac{3}{2}\right) \cup (3, \infty)}$

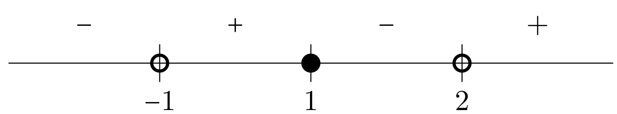
$\square$

(6) Solve $\frac{x-1}{x^2-x-2} \geq 0$

Solution

$$\frac{x-1}{x^2-x-2} \geq 0 \Leftrightarrow \frac{x-1}{(x-2)(x+1)} > 0$$

The numbers to put on our number line are therefore $x = 1, 2, -1$. Testing the intervals we have:



We want the parts that are ≥ 0, so $\boxed{(-1, 1] \cup (2, \infty)}$

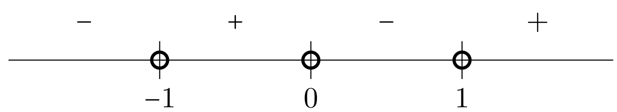
$\square$

(7) Solve $x < \frac{1}{x}$

Solution

$$\begin{aligned} x < \frac{1}{x} &\Leftrightarrow x - \frac{1}{x} < 0 \\ &\Leftrightarrow \frac{x}{x} \cdot x - \frac{1}{x} < 0 \\ &\Leftrightarrow \frac{x^2}{x} - \frac{1}{x} < 0 \\ &\Leftrightarrow \frac{x^2 - 1}{x} < 0 \\ &\Leftrightarrow \frac{(x-1)(x+1)}{x} < 0 \end{aligned}$$

The numbers to put on our number line are therefore $x = 1, -1, 0$. Testing the intervals we have:



We want the parts that are < 0 (negative), so $\boxed{(-\infty, -1) \cup (0, 1)}$

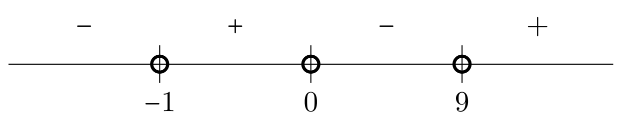
□

(8) Solve $\frac{x^2 - 8x - 9}{x} < 0$

Solution

$$\frac{x^2 - 8x - 9}{x} < 0 \Leftrightarrow \frac{(x-9)(x+1)}{x} < 0$$

The numbers to put on our number line are therefore $x = 9, -1, 0$. Testing the intervals we have:



We want the parts that are < 0 (negative), so $\boxed{(-\infty, -1) \cup (0, 9)}$

□

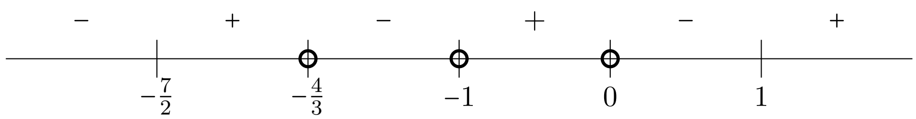
(9) Solve $\frac{2x^3+5x^2-7x}{3x^2+7x+4} > 0$

Solution

$$\frac{2x^3+5x^2-7x}{3x^2+7x+4} > 0 \Leftrightarrow \frac{x(2x^2+5x-7)}{(3x+4)(x+1)} > 0$$

$$\Leftrightarrow \frac{x(2x+7)(x-1)}{(3x+4)(x+1)} > 0$$

The numbers to put on our number line are therefore $x = 0, -\frac{7}{2}, 1, -\frac{4}{3}, -1$. Testing the intervals we have:



We want the parts that are > 0 (positive), so $\boxed{\left(-\frac{7}{2}, -\frac{4}{3}\right) \cup (-1, 0) \cup (1, \infty)}$

□

(10) Solve $1 + \frac{1}{x} \geq \frac{1}{x+1}$

Solution The LCD is $x(x+1)$

$$1 + \frac{1}{x} \geq \frac{1}{x+1} \Leftrightarrow 1 + \frac{1}{x} - \frac{1}{x+1} \geq 0$$

$$\Leftrightarrow 1 \cdot \frac{x(x+1)}{x(x+1)} + \frac{1}{x} \cdot \frac{x+1}{x+1} - \frac{1}{x+1} \cdot \frac{x}{x} \geq 0$$

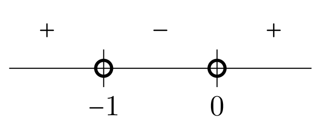
$$\Leftrightarrow \frac{x(x+1)}{x(x+1)} + \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} \geq 0$$

$$\Leftrightarrow \frac{x(x+1) + x + 1 - x}{x(x+1)} \geq 0$$

$$\Leftrightarrow \frac{x^2 + x + x + 1 - x}{x(x+1)} \geq 0$$

$$\Leftrightarrow \frac{x^2 + x + 1}{x(x+1)} \geq 0$$

The $b^2 - 4ac$ for the numerator is negative, so there are no real roots. The only numbers we need for the number line are therefore $x = 0, -1$. Testing the intervals we have:



We want the parts that are ≥ 0, so $\boxed{(-\infty, -1) \cup (0, \infty)}$

□