ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 4.8 Classwork (CW 18) Name: _____

Complete as many of the following problems as you can. You do not have to go in order.

Note: This classwork is optional

(1) Simplify $\sqrt{2}^{1/2} \cdot \sqrt{2}^{3/2}$

Solution

$$\begin{aligned}\sqrt{2}^{1/2} \cdot \sqrt{2}^{3/2} &= \sqrt{2}^{1/2+3/2} \\ &= \sqrt{2}^{4/2} \\ &= \sqrt{2}^{2} \\ &= \boxed{2}\end{aligned}$$

□

(2) Simplify $64^{2/3}$

Solution

$$\begin{aligned}64^{2/3} &= (\sqrt[3]{64})^2 \\ &= 4^2 \\ &= \boxed{16}\end{aligned}$$

□

(3) Simplify $4(2x^{2/3})(7x^{5/4})$

Solution

$$\begin{aligned}4(2x^{2/3})(7x^{5/4}) &= 56x^{2/3+5/4} \\ &= 56x^{8/12+\frac{15}{12}} \\ &= \boxed{56x^{23/12}}\end{aligned}$$

□

(4) Simplify $7^{2/3} \cdot 7^{3/2}$

Solution

$$\begin{aligned} 7^{2/3} \cdot 7^{3/2} &= 7^{2/3+3/2} \\ &= 7^{4/6+9/6} \\ &= \boxed{7^{13/6}} \end{aligned}$$

□

(5) Simplify $\left(4^{-3/5}\right)^{2/3}$

Solution

$$\begin{aligned} \left(4^{-3/5}\right)^{2/3} &= 4^{-3/5 \cdot 2/3} \\ &= \boxed{4^{-2/5}} \end{aligned}$$

□

(6) Simplify $(64x^3)^{1/2}$

Solution

$$\begin{aligned} (64x^3)^{1/2} &= 64^{1/2} \cdot (x^3)^{1/2} \\ &= \sqrt{64} x^{3 \cdot 1/2} \\ &= \boxed{8x^{3/2}} \end{aligned}$$

□

(7) Simplify $(-32x^5y^{10})^{1/5}$

Solution

$$\begin{aligned} (-32x^5y^{10})^{1/5} &= (-32)^{1/5}(x^5)^{1/5}(y^{10})^{1/5} \\ &= \sqrt[5]{-32} x^{5 \cdot 1/5} y^{10 \cdot 1/5} \\ &= -2x^1y^2 \\ &= \boxed{-2xy^2} \end{aligned}$$

□

(8) Solve $x^{3/4} = 8$

Solution

$$\begin{aligned} x^{3/4} = 8 &\Leftrightarrow x = 8^{4/3} \\ &\Leftrightarrow x = (\sqrt[3]{8})^4 \\ &\Leftrightarrow x = 2^4 \\ &= \boxed{16} \end{aligned}$$

□

(9) Solve $x^{5/4} + 36 = 4$

Solution

$$\begin{aligned} x^{5/4} + 36 = 4 &\Leftrightarrow x^{5/4} = -32 \\ &\Leftrightarrow x = (-32)^{4/5} \\ &\Leftrightarrow x = (\sqrt[5]{-32})^4 \\ &\Leftrightarrow x = (-2)^4 \\ &= \boxed{16} \end{aligned}$$

□

(10) Solve $x - 2\sqrt{x} - 9 = 0$

Solution Let $y = \sqrt{x}$

$$\begin{aligned} x - 2\sqrt{x} - 9 = 0 &\Leftrightarrow y^2 - 2y - 9 = 0 \\ &\Leftrightarrow y = \frac{2 \pm \sqrt{4 + 36}}{2} \\ &\Leftrightarrow y = \frac{2 \pm \sqrt{40}}{2} \\ &\Leftrightarrow y = \frac{2 \pm 2\sqrt{10}}{2} \\ &\Leftrightarrow y = 1 \pm \sqrt{10} \\ &\Leftrightarrow y = 1 + \sqrt{10}, 1 - \sqrt{10} \\ &\Leftrightarrow \sqrt{x} = 1 + \sqrt{10}, 1 - \sqrt{10} \\ &\Leftrightarrow x = (1 + \sqrt{10})^2, (1 - \sqrt{10})^2 \\ &\Leftrightarrow x = 11 + 2\sqrt{10}, 11 - 2\sqrt{10} \end{aligned}$$

Checking the solutions, only $\boxed{x = 11 + 2\sqrt{10}}$ works

□

(11) Solve $x^{2/3} - 3x^{1/3} - 10 = 0$

Solution Let $y = x^{1/3}$

$$\begin{aligned} x^{2/3} - 3x^{1/3} - 10 = 0 &\Leftrightarrow y^2 - 3y - 10 = 0 \\ &\Leftrightarrow (y-5)(y+2) = 0 \\ &\Leftrightarrow y = 5, -2 \\ &\Leftrightarrow x^{1/3} = 5, -2 \\ &\Leftrightarrow x = 5^3, (-2)^3 \\ &\Leftrightarrow \boxed{x = 125, -8} \end{aligned}$$

□

(12) Solve $3x^{-2} + 7x^{-1} - 6 = 0$

Solution Let $y = x^{-1}$

$$\begin{aligned} 3x^{-2} + 7x^{-1} - 6 = 0 &\Leftrightarrow 3y^2 + 7y - 6 = 0 \\ &\Leftrightarrow (3y-2)(y+3) = 0 \\ &\Leftrightarrow y = \frac{2}{3}, -3 \\ &\Leftrightarrow x^{-1} = \frac{2}{3}, -3 \\ &\Leftrightarrow \frac{1}{x} = \frac{2}{3}, -3 \\ &\Leftrightarrow \boxed{x = \frac{3}{2}, -\frac{1}{3}} \end{aligned}$$

□

(13) Solve $\sqrt{3x+1} = 4$

Solution

$$\begin{aligned} \sqrt{3x+1} = 4 &\Leftrightarrow 3x + 1 = 16 \\ &\Leftrightarrow 3x = 15 \\ &\Leftrightarrow \boxed{x = 5} \end{aligned}$$

(Remember you need to check that these are actual solutions for radical equations)

□

(14) Solve $\sqrt{x-3} = x-5$

Solution

$$\begin{aligned}\sqrt{x-3} = x-5 &\Leftrightarrow x-3 = (x-5)^2\\ &\Leftrightarrow x-3 = x^2-10x+25\\ &\Leftrightarrow x^2-11x+28 = 0\\ &\Leftrightarrow (x-7)(x-4) = 0\\ &\Leftrightarrow x = 7, 4\end{aligned}$$

Checking the solutions, only $\boxed{x=7}$ works

□

(15) Solve $2\sqrt{2x+5} - x = 4$

Solution

$$\begin{aligned}2\sqrt{2x+5} - x = 4 &\Leftrightarrow 2\sqrt{2x+5} = 4+x\\ &\Leftrightarrow (2\sqrt{2x+5})^2 = (4+x)^2\\ &\Leftrightarrow 4(2x+5) = x^2+8x+16\\ &\Leftrightarrow 8x+20 = x^2+8x+16\\ &\Leftrightarrow x^2-4 = 0\\ &\Leftrightarrow (x-2)(x+2) = 0\\ &\Leftrightarrow \boxed{x=-2,2}\end{aligned}$$

(Remember you need to check that these are actual solutions for radical equations)

□

(16) Solve $\sqrt[3]{4x^2+7} - 2 = 0$

Solution

$$\begin{aligned}\sqrt[3]{4x^2+7} - 2 = 0 &\Leftrightarrow \sqrt[3]{4x^2+7} = 2\\ &\Leftrightarrow 4x^2+7 = 2^3\\ &\Leftrightarrow 4x^2+7 = 8\\ &\Leftrightarrow 4x^2-1 = 0\\ &\Leftrightarrow (2x-1)(2x+1) = 0\\ &\Leftrightarrow \boxed{x = \frac{1}{2}, -\frac{1}{2}}\end{aligned}$$

(Remember you need to check that these are actual solutions for radical equations)

□

(17) Solve $\sqrt{x+2} - \sqrt{x} = 1$

Solution

$$\begin{aligned}
\sqrt{x+2} - \sqrt{x} = 1 &\Leftrightarrow \sqrt{x+2} = 1 + \sqrt{x} \\
&\Leftrightarrow x + 2 = \left(1 + \sqrt{x}\right)^2 \\
&\Leftrightarrow x + 2 = 1 + 2\sqrt{x} + x \\
&\Leftrightarrow 1 = 2\sqrt{x} \\
&\Leftrightarrow 1^2 = \left(2\sqrt{x}\right)^2 \\
&\Leftrightarrow 1 = 4x \\
&\Leftrightarrow \boxed{x = \frac{1}{4}}
\end{aligned}$$

(Remember you need to check that these are actual solutions for radical equations)

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