ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 5.1 Classwork (CW 19) Name: ______________________

Complete as many of the following problems as you can. You do not have to go in order.

Note: This classwork is optional

(1) If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find the following:

(a) $f \circ g$ (b) $g \circ f$

Solution

(a)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x-3) \\ &= (2x-3)^2 + 2(2x-3) - 1 \\ &= 4x^2 - 12x + 9 + 4x - 6 - 1 \\ &= \boxed{4x^2 - 8x + 2}\end{aligned}$$

(b)

$$\begin{aligned}(g \circ f)(x) &= g(x^2 + 2x - 1) \\ &= 2(x^2 + 2x - 1) - 3 \\ &= 2x^2 + 4x - 2 - 3 \\ &= \boxed{2x^2 + 4x - 5}\end{aligned}$$

□

(2) Find $f+g$, $f-g$, fg and $\frac{f}{g}$ for each pairs of functions.

(a) $f(x) = 3x^2 + 1,\ g(x) = x + 3$

(b) $f(x) = x^2 + 13x + 42,\ g(x) = x + 6$

Solution

(a)

$$f+g = (3x^2+1) + (x+3) = \boxed{3x^2 + x + 4}$$

$$f-g = (3x^2+1) - (x+3) = 3x^2 + 1 - x - 3 = \boxed{3x^2 - x - 2}$$

$$fg = (3x^2+1)(x+3) = \boxed{2x^3 + 9x^2 + x + 3}$$

$$\frac{f}{g} = \boxed{\frac{3x^2+1}{x+3}}$$

(b)

$$f+g = (x^2+13x+42) + (x+6) = \boxed{x^2 + 14x + 48}$$

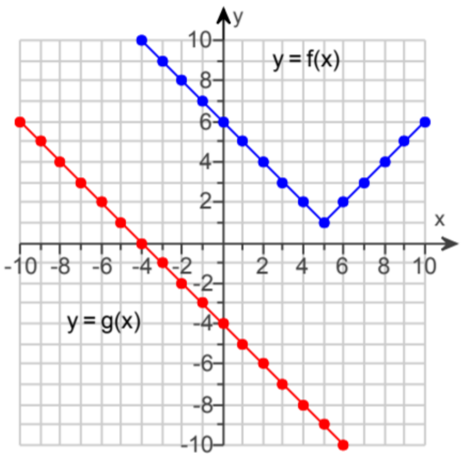
$$f-g = (x^2+13x+42) - (x+6) = x^2 + 13x + 42 - x - 6 = \boxed{x^2 + 12x + 36}$$

$$fg = (x^2+13x+42)(x+6) = x^3 + 6x^2 + 13x^2 + 78x + 42x + 252 = \boxed{x^3 + 19x^2 + 120x + 252}$$

$$\frac{f}{g} = \frac{x^2+13x+42}{x+6} = \frac{(x+7)\cancel{(x+6)}}{\cancel{x+6}} = \boxed{x + 7, x \neq -6}$$

$\square$

(3) Consider the following graph of f and g.



(a) Find $(f \circ g)(0)$

(b) Find $(f \circ g)(-6)$

(c) Find $(g \circ f)(3)$

(d) Find $(g \circ f)(6)$

(e) Find $(f \circ f)(0)$

Solution

(a)

$$(f \circ g)(0) = f(g(0)) = f(-4) = \boxed{10}$$

(b)

$$(f \circ g)(-6) = f(g(-6)) = f(2) = \boxed{4}$$

(c)

$$(g \circ f)(3) = g(f(3)) = g(3) = \boxed{-7}$$

(d)

$$(g \circ f)(6) = g(f(6)) = g(2) = \boxed{-6}$$

(e)

$$(f \circ f)(0) = f(f(0)) = f(6) = \boxed{2}$$

□

(4) Find $(f \circ g)(x)$ and $(g \circ f)(x)$. What is $(f \circ g)(3)$ and $(g \circ f)(3)$?

(a) $f(x) = 2x,\ g(x) = x - 3$

(b) $f(x) = \dfrac{4}{x+3},\ g(x) = \dfrac{1}{x}$

Solution

(a)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x+5) \\ &= 4(x+5) \\ &= \boxed{4x+20}\end{aligned}$$

$$\begin{aligned}(f \circ g)(3) &= 4(3) + 20 \\ &= 12 + 20 \\ &= \boxed{32}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(4x) \\ &= \boxed{4x+5}\end{aligned}$$

$$\begin{aligned}(g \circ f)(3) &= 4(3) + 5 \\ &= 12 + 5 \\ &= \boxed{17}\end{aligned}$$

(b)

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f\left(\frac{1}{x}\right) \\
&= \frac{4}{\frac{1}{x} + 3} \\
&= \boxed{\frac{4x}{1 + 3x}, x \neq 0}
\end{aligned}
$$

$$
\begin{aligned}
(f \circ g)(3) &= \frac{4(3)}{1 + 3(3)} \\
&= \frac{12}{1 + 9} \\
&= \frac{12}{10} \\
&= \boxed{\frac{6}{5}}
\end{aligned}
$$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= g\left(\frac{4}{x + 3}\right) \\
&= \frac{1}{\frac{4}{x+3}} \\
&= \boxed{\frac{x + 3}{4}}
\end{aligned}
$$

$$
\begin{aligned}
(g \circ f)(3) &= \frac{3 + 3}{4} \\
&= \frac{6}{4} \\
&= \boxed{\frac{3}{2}}
\end{aligned}
$$

□

(5) Find two (non-identity) functions f and g such that $h(x) = (f \circ g)(x)$, where

(a) $h(x) = (2x - 1)^8$

(b) $h(x) = \sqrt[3]{3x - 4}$

(c) $h(x) = \sqrt{3x^3 - 4x + 7}$

(d) $h(x) = \dfrac{1}{2x - 3} + 1$

(e) $h(x) = |4x + 5|$

Solution Note: There are multiple possible answers for these.

(a) $g(x) = 2x - 1,\ f(x) = x^8$

(b) $g(x) = 3x - 4,\ f(x) = \sqrt[3]{x}$

(c) $g(x) = 3x^3 - 4x + 7,\ g(x) = \sqrt{x}$

(d) $g(x) = 2x - 3,\ f(x) = \dfrac{1}{x} + 2$

(e) $g(x) = 4x + 5,\ f(x) = |x|$

□