ACMAT117 Fall 2024
Professor Manguba-Glover
Section 1.4 Classwork (CW 3) Name: ______________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each group will be given a specific problem that they must complete and present to either Professor MG or to Stefanie before they leave.

If **your entire table** finishes early, and you have presented your given problem, you may leave early.

(1) Find the slope of the line passing through the points below, or state that the slope undefined.

(a) $(5,3)$ and $(6,8)$

(b) $(-5,1)$ and $(5,5)$

(c) $(5,7)$ and $(6,9)$

(d) $(-1,2)$ and $(5,6)$

Solution

(a)

$$\begin{aligned} m &= \frac{8-3}{6-5} \\ &= \frac{5}{1} \\ &= \boxed{5} \end{aligned}$$

(b)

$$\begin{aligned} m &= \frac{5-1}{5-(-5)} \\ &= \frac{4}{5+5} \\ &= \frac{4}{10} \\ &= \boxed{\frac{2}{5}} \end{aligned}$$

(c)

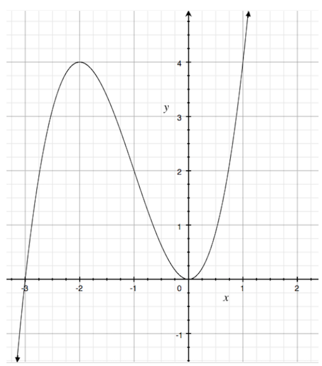
$$\begin{aligned} m &= \frac{9-7}{6-5} \\ &= \frac{2}{1} \\ &= \boxed{2} \end{aligned}$$

(d)

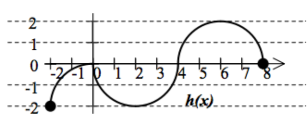
$$\begin{aligned} m &= \frac{6-2}{5-(-1)} \\ &= \frac{4}{5+1} \\ &= \frac{4}{6} \\ &= \boxed{\frac{2}{3}} \end{aligned}$$

□

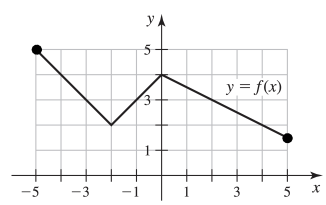
(2) Use interval notation to write where the function is increasing and where it is decreasing, then give the coordinates of the x and y intercepts.



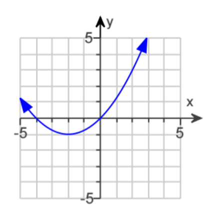
(a)



(b)



(c)



(d)

Solution Functions are increasing when they're heading up and to the right. They are decreasing if they're heading down. The x-intercepts are where the graph touches the x-axis, and the y-intercept is where the graph touches the y-axis.

(a) Increasing on $(-\infty,-2)$, $(0,\infty)$
Decreasing on $(-2,0)$
y-intercept: 0 or $(0,0)$
x-intercept: 0 or $(0,0)$

(b) Increasing on $(-2,0)$, $(2,6)$
Decreasing on $(0,2)$, $(6,8)$
y-intercept: 0 or $(0,0)$
x-intercepts: 0, 4, and 8
or $(0,0)$, $(4,0)$, and $(8,0)$

(c) Increasing on $(-2,0)$
Decreasing on $(-5,-2)$, $(0,5)$
y-intercept: 4 or $(0,4)$
x-intercept: none

(d) Increasing on $(-2,\infty)$
Decreasing on $(-\infty,-2)$
y-intercept: 0 or $(0,0)$
x-intercepts: -4 and 0 or $(-4,0)$, $(0,0)$

□

(3) Calculate the average rate of change of $f(x) = 2x + 10$ as x changes from 3 to 7

Solution

$$\begin{aligned}
\text{Avg rate of change} &= \frac{f(7) - f(3)}{7 - 3} \\
&= \frac{[2(7) + 10] - [2(3) + 10]}{4} \\
&= \frac{(14 + 10) - (6 + 10)}{4} \\
&= \frac{24 - 16}{4} \\
&= \frac{8}{4} \\
&= \boxed{2}
\end{aligned}$$

□

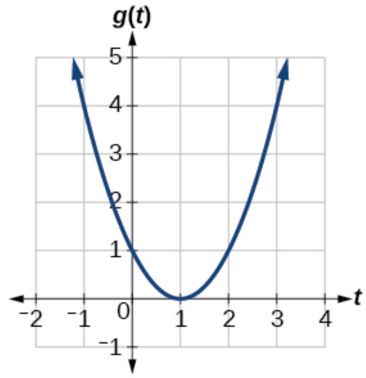
(4) Find the average rate of change of the function $f(x) = x^2 - 5x$ on the interval $4 \leq x \leq 8$ (i.e. from $x = 4$ to $x = 8$)

Solution

$$\begin{aligned}
\text{Avg rate of change} &= \frac{f(8) - f(4)}{8 - 4} \\
&= \frac{[8^2 - 5(8)] - [4^2 - 5(4)]}{4} \\
&= \frac{(64 - 40) - (16 - 20)}{4} \\
&= \frac{24 - (-4)}{4} \\
&= \frac{28}{4} \\
&= \boxed{7}
\end{aligned}$$

□

(5) Using the graph of the function $g(t)$ below, find the average rate of change on the interval $[-1, 2]$ (i.e. from $t = -1$ to $= 2$)



Solution Remember that $f(x)$ is equivalent to the y-value on the graph.

$$\text{Avg rate of change} = \frac{f(2) - f(-1)}{2 - (-1)}$$
$$= \frac{1 - 4}{2 + 1}$$
$$= \frac{-3}{3}$$
$$= \boxed{-1}$$

□

(6) Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = 2x + 7$ (assuming $h \neq 0$)

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{[2(x+h) + 7] - (2x + 7)}{h}$$
$$= \frac{\cancel{2x} + 2h + \cancel{7} - \cancel{2x} - \cancel{7}}{h}$$
$$= \frac{2\cancel{h}}{\cancel{h}}$$
$$= 2$$

□

(7) Find $\frac{f(x+h)-f(x)}{h}$ for $f(x)=2x^2+3x-1$ (assuming $h\neq 0$)

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{(2(x+h)^2+3(x+h)-1)-(2x^2+3x-1)}{h}\\
&= \frac{(2(x^2+2xh+h^2)+3x+3h-1)-(2x^2+3x-1)}{h}\\
&= \frac{(2x^2+4xh+2h^2+3x+3h-1)-(2x^2+3x-1)}{h}\\
&= \frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h-\cancel{1}-\cancel{2x^2}-\cancel{3x}+\cancel{1}}{h}\\
&= \frac{4xh+2h^2+3h}{h}\\
&= \boxed{4x+2h+3}
\end{aligned}$$

□

Key:

(1)
- (a) 5
- (b) $\frac{2}{5}$
- (c) 2
- (d) $\frac{2}{3}$

(2)
- (a) Inc: $(-\infty,-2)$, $(0,\infty)$
  Dec: $(-2,0)$
  x-int: $(-3,0)$ and $(0,0)$
  y-int: $(0,0)$
- (b) Inc: $(-2,0)$, $(2,6)$
  Dec: $(0,2)$, $(6,8)$
  x-int: $(0,0)$, $(4,0)$, $(8,0)$
  y-int: $(0,0)$
- (c) Inc: $(-2,0)$
  Dec: $(-5,-2)$, $(0,5)$
  x-int: none
  y-int: $(0,4)$
- (d) Inc: $(-2,\infty)$
  Dec: $(-\infty,-2)$
  x-int: $(-4,0)$, $(0,0)$
  y-int: $(0,0)$

(3) 2

(4) 7

(5) −1

(6) 2

(7) $4x+2h+3$