

Complete as many of the following problems as you can with your group. You do not have to go in order. Each group will be given a specific problem that they must complete and present to either Professor MG or to Stefanie before they leave.

If **your entire table** finishes early, and you have presented your given problem, you may leave early.

(1) Solve the following linear equations, then check your answers:

(a) $2x + 9 = 14$

(c) $\frac{x-3}{4} = \frac{5}{14} - \frac{x+5}{7}$

(b) $-2b + 8 = 3b - 7$

Solution

(a)

$$\begin{aligned} 2x + 9 = 14 &\Leftrightarrow 2x = 5 \\ &\Leftrightarrow x = \frac{5}{2} \end{aligned}$$

(b)

$$\begin{aligned} -2b + 8 = 3b - 7 &\Leftrightarrow 5b = 15 \\ &\Leftrightarrow \boxed{b = 3} \end{aligned}$$

(c)

$$\begin{aligned} \frac{x-3}{4} = \frac{5}{14} - \frac{x+5}{7} &\Leftrightarrow 28\left(\frac{x-3}{4}\right) = 28\left(\frac{5}{14} - \frac{x+5}{7}\right) \\ &\Leftrightarrow 7(x-3) = 2(5) - 4(x+5) \\ &\Leftrightarrow 7x - 21 = 10 - 4x - 20 \\ &\Leftrightarrow 7x - 21 = -4x - 10 \\ &\Leftrightarrow 11x = 11 \\ &\Leftrightarrow x = 1 \end{aligned}$$

□

(2) Solve the following linear equations, then check your answers:

(a) $4(2x - 4) = -20$

(c) $\frac{1}{2}(x + 4) = \frac{1}{3}x$

(b) $5 - \frac{2x}{3} = -9$

Solution

(a)

$$4(2x - 4) = -20 \Leftrightarrow 8x - 16 = -20$$

$$\Leftrightarrow 8x = -4$$

$$\Leftrightarrow x = -\frac{4}{8}$$

$$\Leftrightarrow x = -\frac{1}{2}$$

(b)

$$5 - \frac{2x}{3} = -9 \Leftrightarrow -\frac{2x}{3} = -14$$

$$\Leftrightarrow -2x = -42$$

$$\Leftrightarrow \boxed{x = 21}$$

(c)

$$\frac{1}{2}(x + 4) = \frac{1}{3}x \Leftrightarrow \frac{1}{2}x + 2 = \frac{1}{3}x$$

$$\Leftrightarrow 6\left(\frac{1}{2}x + 2\right) = 6\left(\frac{1}{3}x\right)$$

$$\Leftrightarrow 3x + 12 = 2x$$

$$\Leftrightarrow \boxed{x = -12}$$

□

(3) Determine if the following equations are conditional equations, contradictions, or identities:

(a) $5(a - 3) - 3(a - 6) = 2(a + 1) + 1$ (c) $\frac{2-3x}{4} = 5$

(b) $2(3m + 1) = 6m + 3$

Solution

(a)

$$\begin{aligned} 5(a - 3) - 3(a - 6) = 2(a + 1) + 1 &\Leftrightarrow 5a - 15 - 3a + 18 = 2a + 2 + 1 \\ &\Leftrightarrow 2a + 3 = 2a + 3 \\ &\Leftrightarrow 3 = 3 \end{aligned}$$

So this is an identity

(b)

$$\begin{aligned} 2(3m + 1) = 6m + 3 &\Leftrightarrow 6m + 2 = 6m + 3 \\ &\Leftrightarrow 2 = 3 \end{aligned}$$

So this is a contradiction

(c)

$$\begin{aligned} \frac{2-3x}{4} = 5 &\Leftrightarrow 2 - 3x = 20 \\ &\Leftrightarrow -3x = 18 \\ &\Leftrightarrow x = -6 \end{aligned}$$

So this is a conditional equation

□

(4) Solve the following linear inequalities. Write your answer in **interval notation**.

(a) $-3x - 1 \geq 11$

(c) $-6x + 4 < -14$

(b) $5x - 7 \geq -17$

(d) $\frac{1}{4}z - \frac{1}{2} < \frac{2z}{3} + 2$

Solution

(a)

$$\begin{aligned} -3x - 1 \geq 11 &\Leftrightarrow -3x \geq 12 \\ &\Leftrightarrow x \leq -4 \\ &\Leftrightarrow \boxed{(-\infty, -4]} \end{aligned}$$

(b)

$$\begin{aligned} 5x - 7 \geq -17 &\Leftrightarrow 5x \geq -10 \\ &\Leftrightarrow x \geq -2 \\ &\Leftrightarrow \boxed{[-2, \infty)} \end{aligned}$$

(c)

$$\begin{aligned} -6x + 4 < -14 &\Leftrightarrow -6x < -18 \\ &\Leftrightarrow x > 3 \\ &\Leftrightarrow \boxed{(3, \infty)} \end{aligned}$$

(d)

$$\begin{aligned} \frac{1}{4}z - \frac{1}{2} < \frac{2z}{3} + 2 &\Leftrightarrow 12\left(\frac{1}{4}z - \frac{1}{2}\right) < 12\left(\frac{2z}{3} + 2\right) \\ &\Leftrightarrow 3z - 6 < 8z + 24 \\ &\Leftrightarrow -5z < 30 \\ &\Leftrightarrow z > -6 \\ &\Leftrightarrow \boxed{(-6, \infty)} \end{aligned}$$

□

(5) Solve the following compound inequalities. Write your answer in **interval notation**.

(a) $-1 \leq 2x + 3 < 11$

(c) $-2 \leq \frac{x}{3} + 5 < 4$

(b) $-5 < 2x - 3 < 5$

(d) $15 \leq 7 - \frac{2}{5}x \leq 21$

Solution

(a)

$$-1 \leq 2x + 3 < 11 \Leftrightarrow -4 \leq 2x < 8$$

$$\Leftrightarrow -2 \leq x < 4$$

$$\Leftrightarrow \boxed{[-2, 4)}$$

(b)

$$-5 < 2x - 3 < 5 \Leftrightarrow -2 < 2x < 8$$

$$\Leftrightarrow -1 < x < 4$$

$$\Leftrightarrow \boxed{(-1, 4)}$$

(c)

$$-2 \leq \frac{x}{3} + 5 < 4 \Leftrightarrow -7 \leq \frac{x}{3} < -1$$

$$\Leftrightarrow -21 \leq x < -3$$

$$\Leftrightarrow \boxed{[-21, -3)}$$

(d)

$$15 \leq 7 - \frac{2}{5}x \leq 21 \Leftrightarrow 8 \leq -\frac{2}{5}x \leq 14$$

$$\Leftrightarrow 40 \leq -2x \leq 70$$

$$\Leftrightarrow -20 \geq x \geq -35$$

$$\Leftrightarrow \boxed{[-35, -20]}$$

□

Key:

(1) (a) $x = \frac{5}{2}$

(2) (a) $x = -\frac{1}{2}$

(3) (a) Identity

(4) (a) $(-\infty, -4]$

(5) (a) $[-2, 4)$

(b) $b = 3$

(b) $x = 21$

(b) Contradiction

(b) $[-2, \infty)$

(b) $(-1, 4)$

(c) $x = 1$

(c) $x = -12$

(c) Conditional

(c) $(3, \infty)$

(c) $[-21, -3)$

(d) $(-6, \infty)$

(d) $[-35, -20]$