ACMAT117 Fall 2024
Professor Manguba-Glover
Section 3.1 Classwork (CW 7) Name: ______________________

Complete as many of the following problems as you can with your group. You do not have to go in order. Each group will be given a specific problem that they must complete and present to either Professor MG or to Stefanie before they leave.

If **your entire table** finishes early, and you have presented your given problem, you may leave early.

(1) Find the vertex using BOTH methods, axis of symmetry, and then graph the following parabolas.

(a) $y = 2x^2 + 8x + 3$ (b) $y = x^2 - 4x$

Solution

(a) Method 1: Completing the square

$$\begin{aligned} y = 2x^2 + 8x + 3 &\Leftrightarrow y = 2(x^2 + 4x) + 3 \\ &\Leftrightarrow y = 2\left(x^2 + 4x + \left(\frac{4}{2}\right)^2\right) + 3 - 2\left(\frac{4}{2}\right)^2 \\ &\Leftrightarrow y = 2(x^2 + 4x + 4) + 3 - 2(4) \\ &\Leftrightarrow y = 2(x+2)^2 - 5 \end{aligned}$$

Method 2

$$\begin{aligned} \text{Vertex} &= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\ &= \left(-\frac{-8}{2(2)}, f\left(-\frac{-8}{2(2)}\right)\right) \\ &= \left(-\frac{8}{4}, f\left(-\frac{8}{4}\right)\right) \\ &= (-2, f(-2)) \\ &= (-2, 2(-2)^2 + 8(-2) + 3) \\ &= (-2, 2(4) - 16 + 3) \\ &= (-2, 8 - 16 + 3) \\ &= (-2, -5) \end{aligned}$$

So the vertex is $(-2, -5)$ and the axis of symmetry is $x = -2$

(b) Method 1: Completing the square

$$\begin{aligned} y = x^2 - 4x &\Leftrightarrow y = x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 \\ &\Leftrightarrow y = x^2 - 4x + (-2)^2 - (-2)^2 \\ &\Leftrightarrow y = x^2 - 4x + 4 - 4 \\ &\Leftrightarrow y = (x-2)^2 - 4 \end{aligned}$$

Method 2:

$$\begin{aligned} \text{Vertex} &= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\ &= \left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right)\right) \\ &= \left(\frac{4}{2}, f\left(\frac{4}{2}\right)\right) \\ &= (2, f(2)) \\ &= (2, (2)^2 - 4(2)) \\ &= (2, 4-8) \\ &= (2, -4) \end{aligned}$$

So the vertex is $(2, -4)$ and the axis of symmetry is $x = 2$

□

(2) Find the vertex using BOTH methods, axis of symmetry, and then graph the following parabolas.

(a) $y = 1 - x^2$

(b) $y = x^2 - 2x - 3$

Solution

(a) Method 1: Completing the square

$$\begin{aligned} y = 1 - x^2 &\Leftrightarrow y = -x^2 + 1 \\ &\Leftrightarrow -(x^2) + 1 \\ &\Leftrightarrow -(x-0)^2 + 1 \end{aligned}$$

Method 2:

$$\begin{aligned} \text{Vertex} &= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\ &= \left(-\frac{0}{2(-1)}, f\left(-\frac{0}{2}\right)\right) \\ &= (0, f(0) \\ &= (0, 1 - 0^2) \\ &= (0, 1) \end{aligned}$$

So the vertex is $(0, 1)$ and the axis of symmetry is $x = 0$

(b) Method 1: Completing the square

$$\begin{aligned} y = x^2 - 2x - 3 &\Leftrightarrow y = \left(x^2 - 2x + \left(\frac{-2}{2}\right)^2\right) - 3 - \left(\frac{-2}{2}\right)^2 \\ &\Leftrightarrow y = (x^2 - 2x + (-1)^2) - 3 - (-1)^2 \\ &\Leftrightarrow y = (x^2 - 2x + 1) - 3 - 1 \\ &\Leftrightarrow y = (x-1)^2 - 4 \end{aligned}$$

Method 2:

$$\begin{aligned}
\text{Vertex} &= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\
&= \left(-\frac{-2}{2(1)}, f\left(\frac{-2}{2(1)}\right)\right) \\
&= \left(\frac{2}{2}, f\left(\frac{2}{2}\right)\right) \\
&= (1, f(1)) \\
&= (1, (1)^2 - 2(1) - 3) \\
&= (1, 1 - 2 - 3) \\
&= (1, -4)
\end{aligned}$$

So the vertex is $(1, -4)$ and the axis of symmetry is $x = 1$

$\square$

(3) Find the vertex using BOTH methods, axis of symmetry, and then graph the following parabolas.

(a) $y = -x^2 + 6x + 2$ (b) $y = 2x^2 - 2x$

Solution

(a) <u>Method 1: Completing the square</u>

$$\begin{aligned} y = -x^2 + 6x + 2 &\Leftrightarrow y = -(x^2 - 6x) + 2 \\ &\Leftrightarrow y = -\left(x^2 - 6x + \left(\frac{-6}{2}\right)^2\right) + 2 - (-1)\left(\frac{-6}{2}\right)^2 \\ &\Leftrightarrow y = -(x^2 - 6x + 9) + 2 + 9 \\ &\Leftrightarrow y = -(x-3)^2 + 11 \end{aligned}$$

<u>Method 2:</u>

$$\begin{aligned} \text{Vertex} &= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\ &= \left(-\frac{6}{2(-1)}, f\left(-\frac{6}{2(-1)}\right)\right) \\ &= \left(-\frac{6}{-2}, f\left((-\frac{6}{-2}\right)\right) \\ &= (3, f(3)) \\ &= (3, -(3)^2 + 6(3) + 2 \\ &= (3, -9 + 18 + 2) \\ &= (3, 11) \end{aligned}$$

So the vertex is $(3, 11)$ and the axis of symmetry is $x = 3$

(b) <u>Method 1: Completing the square</u>

$$\begin{aligned} y = 2x^2 - 2x &\Leftrightarrow y = 2(x^2 - x) \\ &\Leftrightarrow y = 2\left(x^2 - x + \left(\frac{-1}{2}\right)^2\right) - 2\left(\frac{-1}{2}\right)^2 \\ &\Leftrightarrow y = 2\left(x^2 - x + \frac{1}{4}\right) - 2\left(\frac{1}{4}\right) \\ &\Leftrightarrow y = 2\left(x - \frac{1}{2}\right)^2 - \frac{1}{2} \end{aligned}$$

Method 2:

$$\begin{aligned}
\text{Vertex} &= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\
&= \left(-\frac{-2}{2(2)}, f\left(-\frac{-2}{2(2)}\right)\right) \\
&= \left(\frac{2}{4}, f\left(\frac{2}{4}\right)\right) \\
&= \left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) \\
&= \left(\frac{1}{2}, 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)\right) \\
&= \left(\frac{1}{2}, 2\left(\frac{1}{4}\right) - 1\right) \\
&= \left(\frac{1}{2}, \frac{1}{2} - 1\right) \\
&= \left(\frac{1}{2}, \frac{1}{2} - \frac{2}{2}\right) \\
&= \left(\frac{1}{2}, -\frac{1}{2}\right)
\end{aligned}$$

So the vertex is $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and the axis of symmetry is $x = \frac{1}{2}$

□

Key:

(1) (a) $(-2, -5)$, $x = -2$
(b) $(2, -4)$, $x = 2$

(2) (a) $(0, 1)$, $x = 0$
(b) $(1, -4)$, $x = 1$

(3) (a) $(3, 11)$, $x = 3$
(b) $\left(\frac{1}{2}, -\frac{1}{2}\right)$, $x = \frac{1}{2}$