ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 3.1-3.2 Classwork (CW 8) Name: _____

Complete as many of the following problems as you can with your group. You do not have to go in order. Each group will be given a specific problem that they must complete and present to either Professor MG or to Stefanie before they leave.

If **your entire table** finishes early, and you have presented your given problem, you may leave early.

(1) Suppose you have 20 ft of fencing that you want to use for a rectangular garden. What dimensions will give you the largest area of space to use?

(2) A baseball is hit straight up with initial velocity of 80 ft/sec at an initial height of 3 ft. The height, s, of the baseball after t seconds can be found using the following equation:

$$s(t) = -16t^2 + 80t + 3$$

(a) How high is the baseball after 2 seconds?

(b) What is the maximum height of the baseball and when does this occur?

Solution

(a)

$$\begin{aligned} s(2) &= -16(2)^2 + 80(2) + 3 \\ &= -16(4) + 160 + 3 \\ &= -64 + 160 + 3 \\ &= \boxed{99 \text{ ft}} \end{aligned}$$

(b) The graph is an upside down parabola, so the maximum height occurs at the vertex

$$\begin{aligned} \text{Vertex } x\text{-value} &= -\frac{b}{2a} \\ &= -\frac{80}{2(-16)} \\ &= \frac{-80}{-32} \\ &= \frac{5}{2} \end{aligned}$$

So, the maximum height occurs at $\frac{5}{2}$ seconds, or $\boxed{2.5 \text{ seconds}}$. The y-value will give us what the maximum height is.

$$\begin{aligned} f\left(\frac{5}{2}\right) &= -16\left(\frac{5}{2}\right)^2 + 80\left(\frac{5}{2}\right) + 3 \\ &= -16\left(\frac{25}{4}\right) + 200 + 3 \\ &= -100 + 200 + 3 \\ &= \boxed{103 \text{ ft}} \end{aligned}$$

□

(3) Solve the following:

(a) $x^2 + 13x + 40 = 0$

(b) $36x^2 - 25 = 0$

(c) $-2x^2 = 5x + 3$

(d) $3x^2 - 30x + 75 = 0$

Solution Note: you can solve these using any of the methods that we used in class. I'm only going to show one.

(a)

$$\begin{aligned} x^2 + 13x + 40 = 0 &\Leftrightarrow (x+8)(x+5) = 0 \\ &\Leftrightarrow x + 8 = 0 \text{ or } x + 5 = 0 \\ &\Leftrightarrow \boxed{x = -8, -5} \end{aligned}$$

(b)

$$\begin{aligned} 36x^2 - 25 = 0 &\Leftrightarrow (6x-5)(6x+5) = 0 \\ &\Leftrightarrow 6x - 5 = 0 \text{ or } 6x + 5 = 0 \\ &\Leftrightarrow 6x = 5 \text{ or } 6x = -5 \\ &\Leftrightarrow \boxed{x = \frac{5}{6} \text{ or } x = -\frac{5}{6}} \end{aligned}$$

(c)

$$\begin{aligned} -2x^2 = 5x + 3 &\Leftrightarrow 2x^2 + 5x + 3 = 0 \\ &\Leftrightarrow (2x+3)(x+1) = 0 \\ &\Leftrightarrow 2x + 3 = 0 \text{ or } x + 1 = 0 \\ &\Leftrightarrow \boxed{x = -\frac{3}{2} \text{ or } x = -1} \end{aligned}$$

(d)

$$\begin{aligned} 3x^2 - 30x + 75 = 0 &\Leftrightarrow x^2 - 10x + 25 = 0 \\ &\Leftrightarrow (x-5)(x-5) = 0 \\ &\Leftrightarrow x - 5 = 0 \\ &\Leftrightarrow \boxed{x = 5} \end{aligned}$$

□

(4) Solve the following:

(a) $x^2 - 7x + 6 = 0$

(b) $3x^2 - 6x - 72 = 0$

(c) $3t^2 - 13t + 10 = 0$

(d) $2x^2 = 8x + 2$

Solution

(a)

$$\begin{aligned} x^2 - 7x + 6 = 0 &\Leftrightarrow (x-6)(x-1) = 0 \\ &\Leftrightarrow x - 6 = 0 \text{ or } x - 1 = 0 \\ &\Leftrightarrow \boxed{x = 6 \text{ or } x = 1} \end{aligned}$$

(b)

$$\begin{aligned} 3x^2 - 6x - 72 = 0 &\Leftrightarrow x^2 - 2x - 24 = 0 \\ &\Leftrightarrow (x-6)(x+4) = 0 \\ &\Leftrightarrow x - 6 = 0 \text{ or } x + 4 = 0 \qquad \Leftrightarrow \boxed{x = 6 \text{ or } x = -4} \end{aligned}$$

(c)

$$\begin{aligned} 3t^2 - 13t + 10 = 0 &\Leftrightarrow (3t-10)(t-1) = 0 \\ &\Leftrightarrow 3t - 10 = 0 \text{ or } t - 1 = 0 \\ &\Leftrightarrow \boxed{t = \frac{10}{3} \text{ or } t = 1} \end{aligned}$$

(d)

$$\begin{aligned} 2x^2 = 8x + 2 &\Leftrightarrow 2x^2 - 8x - 2 = 0 \\ &\Leftrightarrow x^2 - 4x - 1 = 0 \\ &\Leftrightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-1)(1)}}{2(1)} \\ &\Leftrightarrow x = \frac{4 \pm \sqrt{16+4}}{2} \\ &\Leftrightarrow x = \frac{4 \pm \sqrt{20}}{2} \\ &\Leftrightarrow x = \frac{4 \pm 2\sqrt{5}}{2} \\ &\Leftrightarrow x = \boxed{2 \pm \sqrt{5}} \end{aligned}$$

□

(5) Solve the following:

(a) $8x^2 + 8x - 30 = 0$

(b) $x^2 - x - 12 = 0$

(c) $6x^2 = 4 + 5x$

(d) $5x^2 + 2x + 6 = 0$

Solution

(a)

$$\begin{aligned} 8x^2 + 8x - 30 = 0 &\Leftrightarrow 4x^2 + 4x - 15 - 0 \\ &\Leftrightarrow (2x+5)(2x-3) = 0 \\ &\Leftrightarrow 2x + 5 = 0 \text{ or } 2x - 3 = 0 \\ &\Leftrightarrow \boxed{x = -\frac{5}{2} \text{ or } x = \frac{3}{2}} \end{aligned}$$

(b)

$$\begin{aligned} x^2 - x - 12 = 0 &\Leftrightarrow (x-4)(x+3) = 0 \\ &\Leftrightarrow x - 4 = 0 \text{ or } x + 3 = 0 \\ &\Leftrightarrow \boxed{x = 4 \text{ or } x = -3} \end{aligned}$$

(c)

$$\begin{aligned} 6x^2 = 4 + 5x &\Leftrightarrow 6x^2 - 5x - 4 = 0 \\ &\Leftrightarrow (3x-4)(2x+1) = 0 \\ &\Leftrightarrow 3x - 4 = 0 \text{ or } 2x + 1 = 0 \\ &\Leftrightarrow \boxed{x = \frac{4}{3} \text{ or } x = -\frac{1}{2}} \end{aligned}$$

(d) Looking at the discriminant $b^2 - 4ac = 2^2 - 4(6)(5) = 4 - 120 = -116$, so there are $\boxed{\text{no real solutions}}$ (the quadratic formula would have the square root of a negative number).

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Key:

(1) 5 ft by 5 ft

(2) (a) 99 ft
(b) 103 ft, 2.5 s

(3) (a) $x = -5, -8$
(b) $x = -\frac{5}{6}, \frac{5}{6}$
(c) $x = -\frac{3}{2}, -1$
(d) $x = 5$

(4) (a) $x = 1, 6$
(b) $x = 6, -4$
(c) $x = \frac{10}{3}, 1$
(d) $2 + \sqrt{5}, 2 - \sqrt{5}$

(5) (a) $x = -\frac{5}{2}, \frac{3}{2}$
(b) $x = -3, 4$
(c) $x = \frac{4}{3}, -\frac{1}{2}$
(d) No real solutions