ACMAT117 Fall 2024
Professor Manguba-Glover
Sections 3.3 Classwork (CW 9) Name: _____

Complete as many of the following problems as you can with your group. You do not have to go in order. Each group will be given a specific problem that they must complete and present to either Professor MG or to Stefanie before they leave.

(1) Simplify and write your final answer in standard form:

(a) $\sqrt{-50} - \sqrt{-8}$

(b) $\sqrt{-3}(\sqrt{-75} - \sqrt{3})$

(c) $\sqrt{-8} - \sqrt{-18} + \sqrt{-32}$

(d) $\sqrt{(3 + \sqrt{-16})(3 - \sqrt{-16})}$

**Solution**

(a)

$$\begin{aligned}\sqrt{-50} - \sqrt{-8} &= i\sqrt{50} - i\sqrt{8} \\ &= i\sqrt{25 \cdot 2} - i\sqrt{4 \cdot 2} \\ &= 5i\sqrt{2} - 2i\sqrt{2} \\ &= \boxed{3i\sqrt{2}}\end{aligned}$$

(b)

$$\begin{aligned}\sqrt{-3}(\sqrt{-75} - \sqrt{3}) &= i\sqrt{3}(i\sqrt{75} - \sqrt{3}) \\ &= i\sqrt{3}(i\sqrt{25 \cdot 3} - \sqrt{3}) \\ &= i\sqrt{3}(5i\sqrt{3} - \sqrt{3}) \\ &= 5i^2(3) - i(3) \\ &= \boxed{-15 - 3i}\end{aligned}$$

(c)

$$\begin{aligned}\sqrt{-8} - \sqrt{-18} + \sqrt{-32} &= i\sqrt{8} - i\sqrt{18} + i\sqrt{32} \\ &= i\sqrt{4 \cdot 2} - i\sqrt{9 \cdot 2} + i\sqrt{16 \cdot 2} \\ &= 2i\sqrt{2} - 3i\sqrt{2} + 4i\sqrt{2} \\ &= \boxed{3i\sqrt{2}}\end{aligned}$$

(d)

$$\begin{aligned}\sqrt{(3+\sqrt{-16})(3-\sqrt{-16})} &= \sqrt{(3+i\sqrt{16})(3-i\sqrt{16})}\\ &= \sqrt{(3+4i)(3-4i)}\\ &= \sqrt{9-16i^2}\\ &= \sqrt{9+16}\\ &= \sqrt{25}\\ &= \boxed{5}\end{aligned}$$

□

(2) Perform the operation and write your final answer in standard form:

(a) $(6-5i)+(14-3i)-(7+i)$

(b) $(19+i)+7i-(3-4i)+2$

(c) $i+3+(i-3)+(3i-1)$

(d) $2-3i+(4i-5i)+6i-(7i-2)$

**Solution**

(a)

$$\begin{aligned}(6-5i)+(14-3i)-(7+i) &= 6-5i+14-3i-7-i\\ &= \boxed{13-9i}\end{aligned}$$

(b)

$$\begin{aligned}(19+i)+7i-(3-4i)+2 &= 19+i+7i-3+4i+2\\ &= \boxed{18+12i}\end{aligned}$$

(c)

$$\begin{aligned}i+3+(i-3)+(3i-1) &= i+3+i-3+3i-1\\ &= \boxed{-1+5i}\end{aligned}$$

(d)

$$\begin{aligned}2-3i+(4i-5i)+6i-(7i-2) &= 2-3i+4i-5i+6i-7i+2\\ &= \boxed{4-5i}\end{aligned}$$

□

(3) Perform the operation and write your final answer in standard form:

(a) $7i(-4-3i)$ (b) $i(4+i)(1+i)$ (c) $(2+3i)(7-2i)$ (d) $(3-8i)(2+7i)$

**Solution**

(a)

$$\begin{aligned} 7i(-4-3i) &= -28i - 21i^2 \\ &= -28i + 21 \\ &= \boxed{21 - 28i} \end{aligned}$$

(b)

$$\begin{aligned} i(4+i)(1+i) &= (4i + i^2)(1+i) \\ &= (4i-1)(1+i) \\ &= 4i + 4i^2 - 1 - i \\ &= -4 - 1 + 3i \\ &= \boxed{-5 + 3i} \end{aligned}$$

(c)

$$\begin{aligned} (2+3i)(7-2i) &= 14 - 4i + 21i - 6i^2 \\ &= 14 + 17i + 6 \\ &= \boxed{20 + 17i} \end{aligned}$$

(d)

$$\begin{aligned} (3-8i)(2+7i) &= 6 + 21i - 16i - 56i^2 \\ &= 6 + 5i + 56 \\ &= \boxed{62 + 5i} \end{aligned}$$

□

(4) Perform the operation and write your final answer in standard form:

(a) $\frac{4-3i}{5+5i}$ (b) $\frac{17-8i}{-5i}$

**Solution**

(a)

$$\begin{aligned}\frac{4-3i}{5+5i} &= \frac{4-3i}{5+5i}\cdot\frac{5-5i}{5-5i}\\ &= \frac{20-20i-15i+15i^2}{25-10i+10i-25i^2}\\ &= \frac{20-35i-15}{25+25}\\ &= \frac{5-35i}{50}\\ &= \frac{5(1-7i)}{50}\\ &= \frac{1-7i}{10}\\ &= \boxed{\frac{1}{10}-\frac{7}{10}i}\end{aligned}$$

(b)

$$\begin{aligned}\frac{17-8i}{-5i} &= \frac{17-8i}{-5i}\cdot\frac{5i}{5i}\\ &= \frac{85i-40i^2}{-25i^2}\\ &= \frac{40+85i}{25}\\ &= \frac{5(8+17i)}{25}\\ &= \frac{8+17i}{5}\\ &= \boxed{\frac{8}{5}+\frac{17}{5}i}\end{aligned}$$

□

(5) Perform the operation and write your final answer in standard form:

(a) $i^{13}$ (b) $-i^{17}$ (c) $(1+i)^3$ (d) $(2i)^5 + i^9$

**Solution**

(a)

$$\begin{aligned} i^{13} &= i^{12} \cdot i \\ &= (i^4)^3 \cdot i \\ &= (1)^3 \cdot i \\ &= \boxed{i} \end{aligned}$$

(b)

$$\begin{aligned} -i^{17} &= -i^{16} \cdot i \\ &= -(i^4)^4 \cdot i \\ &= -(1)^4 \cdot i \\ &= \boxed{-i} \end{aligned}$$

(c)

$$\begin{aligned} (1+i)^3 &= (1+i)(1+i)(1+i) \\ &= (1+2i+i^2)(1+i) \\ &= (1+2i-1)(1+i) \\ &= 2i(1+i) \\ &= 2i+2i^2 \\ &= 2i+2(-1) \\ &= \boxed{-2+2i} \end{aligned}$$

(d)

$$\begin{aligned} (2i)^5 + i^9 &= 2^5 i^5 + i^9 \\ &= 32i^4 \cdot i + i^8 \cdot i \\ &= 32(1)i + (i^4)^2 \cdot i \\ &= 32i + (1)^2 i \\ &= 32i + i \\ &= \boxed{33i} \end{aligned}$$

□

(6) Find the discriminant to determine what kind of solutions the quadratic equation has, then solve the equation.

(a) $x^2 + 11x + 30 = 0$

(b) $2x^2 - 3x - 1 = 0$

(c) $x^2 - 8x + 52 = 0$

(d) $x^2 - 10x + 34 = 0$

**Solution**

(a)

$$\begin{aligned} x^2 + 11x + 30 = 0 &\Leftrightarrow (x+6)(x+5) = 0 \\ &\Leftrightarrow x + 6 = 0 \text{or } x + 5 = 0 \\ &\boxed{x = -6 \text{ or } x = -5} \end{aligned}$$

(b)

$$\begin{aligned} 2x^2 - 3x - 1 = 0 &\Leftrightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)} \\ &\Leftrightarrow x = \frac{3 \pm \sqrt{9+8}}{4} \\ &\Leftrightarrow \boxed{x = \frac{3 \pm \sqrt{17}}{4}} \end{aligned}$$

(c)

$$\begin{aligned} x^2 - 8x + 52 = 0 &\Leftrightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(52)}}{2(1)} \\ &\Leftrightarrow x = \frac{8 \pm \sqrt{64 - 208}}{2} \\ &\Leftrightarrow x = \frac{8 \pm \sqrt{-144}}{2} \\ &\Leftrightarrow x = \frac{8 \pm i\sqrt{144}}{2} \\ &\Leftrightarrow x = \frac{8 \pm 12i}{2} \\ &\Leftrightarrow x = \frac{2(4 \pm 6i)}{2} \\ &\Leftrightarrow x = \boxed{4 \pm 6i} \end{aligned}$$

(d)

$$x^2 - 10x + 34 = 0 \Leftrightarrow x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(34)}}{2(1)}$$

$$\Leftrightarrow x = \frac{10 \pm \sqrt{100 - 136}}{2}$$

$$\Leftrightarrow x = \frac{10 \pm \sqrt{-36}}{2}$$

$$\Leftrightarrow x = \frac{10 \pm i\sqrt{36}}{2}$$

$$\Leftrightarrow x = \frac{10 \pm 6i}{2}$$

$$\Leftrightarrow x = \frac{2(5 \pm 3i)}{2}$$

$$\Leftrightarrow \boxed{x = 5 \pm 3i}$$

□

Key:

(1) (a) $3i\sqrt{2}$
(b) $-15 - 3i$
(c) $3i\sqrt{2}$
(d) 5

(2) (a) $13 - 9i$
(b) $18 + 12i$
(c) $-1 + 5i$
(d) $-5i + 4$

(3) (a) $21 - 28i$
(b) $-5 + 3i$
(c) $20 + 17i$
(d) $62 + 5i$

(4) (a) $\frac{1}{10} - \frac{7}{10}i$
(b) $\frac{8}{5} + \frac{17}{5}i$

(5) (a) $i$
(b) $-i$
(c) $-2 + 2i$
(d) $33i$

(6) (a) $-5, -6$
(b) $\frac{3 \pm \sqrt{17}}{4}$
(c) $4 \pm 6i$
(d) $5 \pm 3i$