

# Reference Sheet

## Fraction Arithmetic

$$\bullet \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \bullet \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

- To add and subtract fractions, you need a common denominator.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \text{and} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

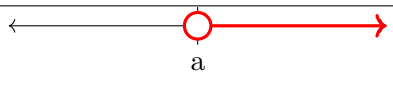
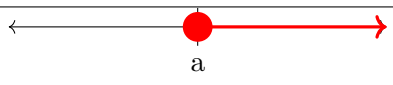
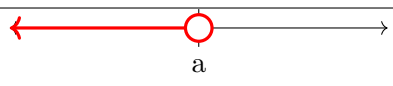
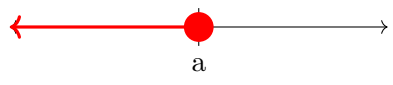
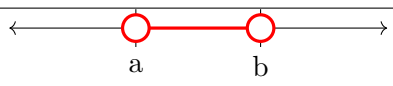
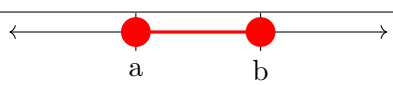
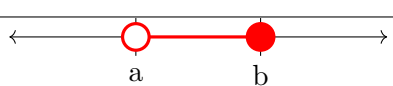
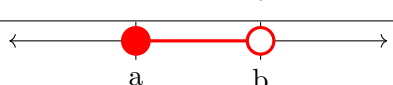
## Miscellaneous

- To convert from a percentage to a decimal, divide the number by 100. This is equivalent to moving the decimal two places to the left. Do the opposite to convert from a decimal to a percentage.
- Distance between two points  $(x_1, y_1)$ ,  $(x_2, y_2)$  is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint between two points  $(x_1, y_1)$ ,  $(x_2, y_2)$  is  $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
- Distance = Rate  $\cdot$  Time (i.e.  $d = rt$ )
- $i = \sqrt{-1}$
- To divide complex numbers, write the division as a fraction and multiply the top and bottom by the conjugate of the denominator

## Relations and Functions

- The domain of a relation/function is the set of all  $x$ -values
- The range of a relation/function is the set of all  $y$ -values
- To find  $x$ -intercepts, set  $y = 0$  and solve. To find a  $y$ -intercept, set  $x = 0$  and solve.
- Average rate of change of a function  $f$  from  $x = a$  to  $x = b$  is  $\frac{f(b)-f(a)}{b-a}$
- Difference quotient of  $f$  is  $\frac{f(x+h)-f(x)}{h}$
- Equation of a circle:  $(x - h)^2 + (y - k)^2 = r^2$ , where the center is  $(h, k)$  and the radius is  $r$

## Inequalities

Inequality	Number Line	Interval Notation
$x > a$		$(a, \infty)$
$x \geq a$		$[a, \infty)$
$x < a$		$(-\infty, a)$
$x \leq a$		$(-\infty, a]$
$a < x < b$		$(a, b)$
$a \leq x \leq b$		$[a, b]$
$a < x \leq b$		$(a, b]$
$a \leq x < b$		$[a, b)$

# Reference Sheet Continued

## Lines/Linear Functions

- Standard/General Form:  $Ax + By + C = 0$ , where  $A$  and  $B$  aren't both 0
- Slope-Intercept Form:  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept
- Point-Slope Form:  $y - y_1 = m(x - x_1)$ , where  $m$  is the slope and the point  $(x_1, y_1)$  is on the line
- $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
- Parallel lines have the same slope. Perpendicular lines have slopes that are opposite reciprocals of each other.

## Quadratic Functions/Inequalities

- Equation of a parabola:  $f(x) = a(x - h)^2 + k$ , where  $(h, k)$  is the vertex.
- The vertex of a parabola  $f(x) = ax^2 + bx + c$  is located at  $(-\frac{b}{2a}, f(-\frac{b}{2a}))$
- To complete the square for  $x^2 + bx$ : add  $(\frac{b}{2})^2$ . If there is a number in front of your  $x^2$ , factor that out before completing the square.
- To factor  $ax^2 + bx + c$  by grouping, find two numbers that multiply to  $a \cdot c$  and add to  $b$ . Use these two numbers to split up the middle term  $bx$ .
- Factor by grouping:  $a(b + c) + d(b + c) = (a + d)(b + c)$
- Difference of squares formula:  $a^2 - b^2 = (a - b)(a + b)$
- Quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $b^2 - 4ac$  is called the discriminant of a quadratic function.

## Transformations of graphs

- To graph a function by applying more than one transformation, (i.e. to graph  $y = af(bx + c) + d$  using  $y = f(x)$ ) use the following order:
  - (1) Horizontal shifts using  $c$  (move left if  $+c$  and right if  $-c$ )
  - (2) Horizontal stretching/shrinking and/or reflecting across  $y$ -axis using  $b$  (divide all  $x$ -values by  $b$ )
  - (3) Vertical stretching/shrinking and/or reflecting across  $x$ -axis using  $a$  (multiply all  $y$ -values by  $a$ )
  - (4) Vertical shifts using  $d$  (move up if  $+d$  and down if  $-d$ )

## Combining Functions

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x) \cdot g(x)$
- $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
- $(f \circ g)(x) = f(g(x))$
- $(g \circ f)(x) = g(f(x))$

## General Functions:

- A function is odd if  $f(-x) = -f(x)$
- A function is even if  $f(-x) = f(x)$
- A turning point is a point where a graph switches from increasing to decreasing or vice versa.
- An inflection point is a point where a graph switches from concave up to concave down or vice versa.
- The leading term of a polynomial is the term of highest degree. The leading coefficient is the coefficient of the leading term.
- A polynomial of degree 0 or 1 is called linear. A polynomial with degree 2 is a quadratic, a polynomial with degree 3 is a cubic, and a polynomial with degree 4 is a quartic.
- The Rational Root Test: If  $x$  is a rational root/zero of a polynomial, then it can be written as  $x = \frac{p}{q}$ , where  $p$  is a factor of the constant term, and  $q$  is a factor of the leading coefficient.
- If  $a + bi$  is a complex root/zero of a polynomial, then so is  $a - bi$  and vice versa.
- The multiplicity of a root/zero is the degree of the factor associated with that root/zero
- A root/zero with an odd multiplicity will have a graph that goes through that number on the  $x$ -axis, a root/zero with even multiplicity will have a graph that bounces off of that number on the  $x$ -axis.
- To solve a polynomial equation:
  - Get everything to one side
  - Factor
  - Set each factor equal to zero and solve
- To solve a polynomial inequality:
  - Get everything to one side
  - Factor
  - Set each factor equal to zero and plot the resulting numbers on a number line
  - Test each number line segment and choose the pieces that satisfy the inequality

## Radicals & Rational Exponents

- $b^n b^m = b^{n+m}$
- $\frac{b^n}{b^m} = b^{n-m}$
- $b^0 = 1$
- $(b^n)^m = b^{nm}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $b^{-n} = \frac{1}{b^n}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- $\sqrt[n]{x} = y \Rightarrow y^n = x$
- $\sqrt[n]{x} = x^{1/n}$
- $x^{m/n} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$
- Solving a radical equation: Isolate the term with the radical (root sign), then raise both sides to the power that matches the radical. Combine like terms and solve.

## Rational Functions

- A rational function is a function of the form  $f(x) = \frac{p(x)}{q(x)}$ , where  $p$  and  $q$  are polynomials
- Vertical asymptotes occur at the  $x$  values where the denominator equals 0
- For horizontal asymptotes:
  - If (degree of numerator) > (degree of denominator), then there is no horizontal asymptote
  - If (degree of numerator) = (degree of denominator), then the horizontal asymptote is  $y = \frac{\text{lead coeff. of num.}}{\text{lead coeff. of denom.}}$
  - If (degree of numerator) < (degree of denominator), then the horizontal asymptote is  $y = 0$
- If the degree of the numerator is exactly one more than the degree of the denominator, there is a slant asymptote. You find the equation of the slant asymptote by doing polynomial division
- Steps to solving a rational equation:
  - (1) Either multiply all values by the least common denominator (LCD) **OR** get everything into one fraction on one side using the LCD
  - (2) Factor if needed
  - (3) Solve for  $x$
- To solve a rational inequality:
  - Get everything to one side
  - Use a common denominator to get everything into one fraction (do NOT multiply by the LCD for inequalities)
  - Set each factor equal to zero and plot the resulting numbers on a number line
  - Test each number line segment and choose the pieces that satisfy the inequality

I have no information to put on this page. I just wanted to give you a reason to have an extra page for writing down example problems :)