Name: _____
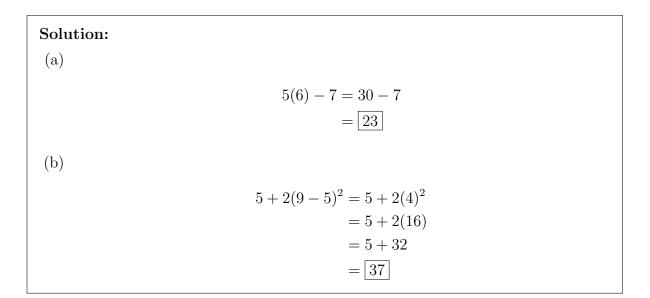
┌───┐
│ Answer the questions in the spaces provided on the question sheets. If you │
│ run out of room for an answer, raise hand for additional paper.   │
│ No calculators or notes allowed.   │
│ Show all work clearly. Use proper notation.   │
└───┘

Question	Points	Score
1	8	
2	7	
3	10	
4	5	
5	15	
6	20	
7	3	
8	15	
9	15	
10	15	
Total	113	

1. (8 points) for each part, evaluate the given algebraic expression for the given values of the variable(s):

   (a) (4 points) $5x - y$, for $x = 6$ and $y = 7$

   (b) (4 points) $5 + 2\left(x - 5\right)^2$, for $x = 9$

Solution:

(a)
$$5(6) - 7 = 30 - 7$$
$$= \boxed{23}$$

(b)
$$5 + 2(9 - 5)^2 = 5 + 2(4)^2$$
$$= 5 + 2(16)$$
$$= 5 + 32$$
$$= \boxed{37}$$

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2. (7 points) The formula $C = \frac{5}{9}(F - 32)$ expresses the relationship between Fahrenheit temperature, F, and Celsius temperature, C. Use the formula to convert $95°F$ to its equivalent temperature on the Celsius scale.

Solution:

$$\frac{5}{9}(95 - 32) = \frac{5}{9}(63)$$
$$= 5(7)$$
$$= \boxed{35°C}$$

3. (10 points) Answer the following about sets.

(a) (5 points) What is $\{1, 7, 8, 9\} \cap \{7, 8, 10\}$?
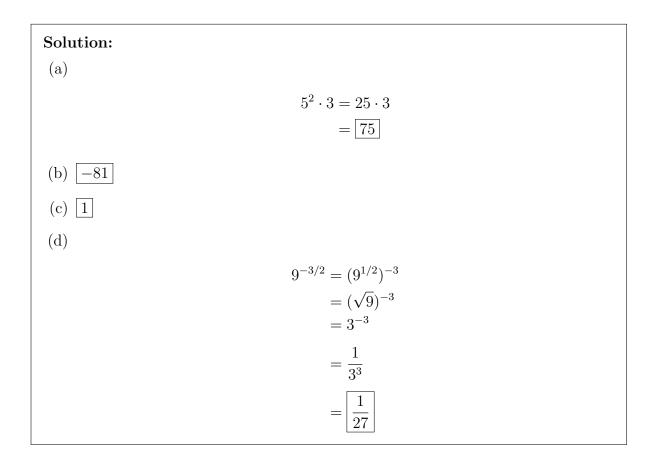
(b) (5 points) What is $\{1, 7, 8, 9\} \cup \{7, 8, 10\}$?

Solution:

(a) $\boxed{\{7, 8\}}$

(b) $\boxed{\{1, 7, 8, 9, 10\}}$

4. (5 points) Rewrite the expression without using absolute value bars.

$$\big||-2| - |-7|\big|$$

Solution:

$$\big||-2| - |-7|\big| = |2 - 7|$$
$$= |-5|$$
$$= \boxed{5}$$

5. (15 points) Evaluate the following:

   (a) (3 points) $5^2 \cdot 3$

   (b) (3 points) -3^4

   (c) (3 points) $(-2)^0$

   (d) (5 points) $9^{-3/2}$

Solution:

(a)
$$5^2 \cdot 3 = 25 \cdot 3$$
$$= \boxed{75}$$

(b) $\boxed{-81}$

(c) $\boxed{1}$

(d)
$$9^{-3/2} = (9^{1/2})^{-3}$$
$$= (\sqrt{9})^{-3}$$
$$= 3^{-3}$$
$$= \frac{1}{3^3}$$
$$= \boxed{\frac{1}{27}}$$

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6. (20 points) Simplify the following expressions. (Remember: simplified expressions have positive exponents)

    (a) (5 points) $\left(x^{-9}\right)^8$

    (b) (5 points) $\left(-2x^4y^5\right)^4$

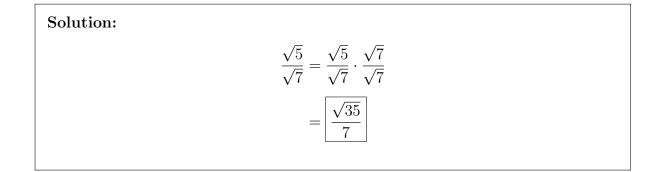   (c) (5 points) $\left(\frac{-45a^2b^9}{15a^6b^{-2}}\right)^3$.

    (d) (5 points) $\dfrac{\sqrt{75x^3}}{\sqrt{3x}}$. Assume that $x > 0$.

Solution:

(a)

$$(x^{-9})^8 = x^{-9 \cdot 8}$$
$$= x^{-72}$$
$$= \boxed{\frac{1}{x^{72}}}$$

(b)

$$(-2x^4 y^5)^4 = (-2)^4 (x^4)^4 (y^5)^4$$
$$= 16 x^{4 \cdot 4} y^{5 \cdot 4}$$
$$= \boxed{16 x^{16} y^{20}}$$

(c)

$$\left(\frac{-45 a^2 b^9}{15 a^6 b^{-2}} \right)^3 = \left(\frac{-45}{15} \cdot \frac{a^2}{a^6} \cdot \frac{b^9}{b^{-2}} \right)^3$$
$$= \left(-3 \cdot \frac{1}{a^4} \cdot b^{11} \right)^3$$
$$= \left(\frac{-3 b^{11}}{a^4} \right)^3$$
$$= \frac{(-3)^3 (b^{11})^3}{(a^4)^3}$$
$$= \frac{-27 b^{11 \cdot 3}}{a^{4 \cdot 3}}$$
$$= \boxed{\frac{-27 b^{33}}{a^{12}}}$$

(d)

$$\frac{\sqrt{75 x^3}}{\sqrt{3x}} = \frac{\sqrt{25 x^3 3x}}{\sqrt{3x}}$$
$$= \frac{5x \sqrt{3x}}{\sqrt{3x}}$$
$$= \boxed{5x}$$

7. (3 points) Rationalize the denominator and simplify the answer.

$$\frac{\sqrt{5}}{\sqrt{7}}$$

Solution:

$$\frac{\sqrt{5}}{\sqrt{7}} = \frac{\sqrt{5}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \boxed{\frac{\sqrt{35}}{7}}$$

8. (15 points) For the following parts, perform the operation on the given polynomials, and state the degree of the resulting polynomial.
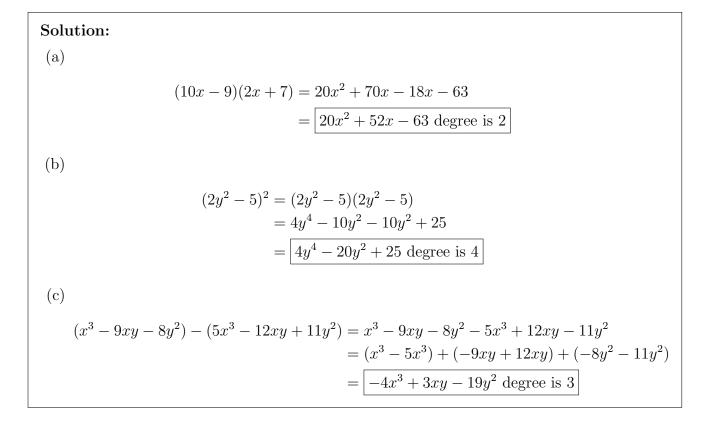
(a) (5 points) Find the product:
$$(10x - 9)(2x + 7)$$

(b) (5 points) Find the product:
$$\left(2y^2 - 5\right)^2$$

(c) (5 points) Subtract the polynomials:
$$(x^3 - 9xy - 8y^2) - (5x^3 - 12xy + 11y^2)$$

Solution:

(a)
$$(10x - 9)(2x + 7) = 20x^2 + 70x - 18x - 63$$
$$= \boxed{20x^2 + 52x - 63 \text{ degree is } 2}$$

(b)
$$(2y^2 - 5)^2 = (2y^2 - 5)(2y^2 - 5)$$
$$= 4y^4 - 10y^2 - 10y^2 + 25$$
$$= \boxed{4y^4 - 20y^2 + 25 \text{ degree is } 4}$$

(c)
$$(x^3 - 9xy - 8y^2) - (5x^3 - 12xy + 11y^2) = x^3 - 9xy - 8y^2 - 5x^3 + 12xy - 11y^2$$
$$= (x^3 - 5x^3) + (-9xy + 12xy) + (-8y^2 - 11y^2)$$
$$= \boxed{-4x^3 + 3xy - 19y^2 \text{ degree is } 3}$$

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9. (15 points) Factor the following polynomials:

   (a) (5 points) $x^2 + 12x + 35$

   (b) (5 points) $10x^2 + 15x$

   (c) (5 points) $x^2 - 36$

Solution:

(a) $\boxed{(x + 7)(x + 5)}$

(b) $\boxed{5x(2x + 3)}$

(c) $\boxed{(x - 6)(x + 6)}$

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10. (15 points) Perform the given operations on the given rational expressions, and state which numbers must be excluded, if any.

   (a) (7 points) Multiply
   $$\frac{x-6}{3x+15} \cdot \frac{4x+20}{4x-24}$$

   (b) (8 points) Add
   $$\frac{4}{x-4} + \frac{5}{x+7}$$

Solution:

(a)

$$\frac{x-6}{3x+15} \cdot \frac{4x+20}{4x-24} = \frac{x-6}{3(x+5)} \cdot \frac{4(x+5)}{4(x-6)}$$

$$= \frac{\cancel{4}}{3 \cdot \cancel{4}}$$

$$= \boxed{\frac{1}{3}, x \neq -5, 6}$$

(b)

$$\frac{4}{x-4} + \frac{5}{x+7} = \frac{4(x+7)+5(x-4)}{(x-4)(x+7)}$$

$$= \frac{4x+28+5x-20}{(x-4)(x+7)}$$

$$= \boxed{\frac{9x+8}{(x-4)(x+7)}, x \neq 4, -7}$$