Read the instructions carefully. Write legibly. Indicate your answers clearly. Show all your work. No notes, calculators, or other electronic devices allowed during the exam.

1. Solve the linear equation

$$6(x-1) = 5x - 90$$

Solution

$$\begin{aligned} 6(x-1) = 5x - 90 &\Leftrightarrow 6x - 6 = 5x - 90 \\ &\Leftrightarrow 6x - 5x = -90 + 6 \\ &\Leftrightarrow \boxed{x = -84} \end{aligned}$$

□

2. Solve the formula for q:

$$aq = h + bq$$

Solution

$$\begin{aligned} aq = h + bq &\Leftrightarrow aq - bq = h \\ &\Leftrightarrow q(a-b) = h \\ &\Leftrightarrow \boxed{q = \frac{h}{a-b}} \end{aligned}$$

□

3. Solve the equation. Then determine whether the equation is an identity, a conditional equation, or an inconsistent equation.

$$3(x-5) - 3x + 5 = 3$$

Solution

$$\begin{aligned} 3(x-5) - 3x + 5 = 3 &\Leftrightarrow \cancel{3x} - 15 - \cancel{3x} + 5 = 3 \\ &\Leftrightarrow -10 = 3 \end{aligned}$$

Thus there is $\boxed{\text{no solution}}$ and it is an $\boxed{\text{inconsistent equation}}$

□

4. Graph the equation

$$y = x^2 - 3$$

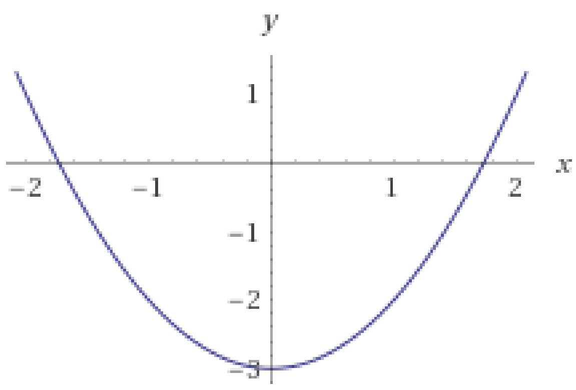
by plotting points on the given cartesian plane. Let $x = -3, -2, -1, 0, 1, 2, 3$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	

Determine the x-intercepts, if any.
Determine the y-intercepts, if any.

Solution

x	y
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



The x-intercepts are when $y = 0$

$$0 = x^2 - 3 \Leftrightarrow x^2 = 3 \Leftrightarrow x = \pm\sqrt{3}$$

So the x-intercepts are $\boxed{\pm\sqrt{3}}$
The y-intercept is when $x = 0$ so the y-intercept is $\boxed{-3}$

□

5. Consider the equation

$$\frac{1}{3x+3}+2=\frac{2}{x+1}$$

a) Write the value(s) of x that make the denominator zero (i.e.: the restrictions).

b) Keeping the restrictions in mind, solve the equation.

Solution

a)

$$3x+3=0 \Leftrightarrow 3x=-3 \Leftrightarrow x=-1$$

$$x+1=0 \Leftrightarrow x=-1$$

So the value that makes the denominators zero is $\boxed{x=-1}$

b)

$$\begin{aligned}
\frac{1}{3x+3}+2=\frac{2}{x+1} &\Leftrightarrow \frac{1}{3(x+1)}+2=\frac{2}{x+1}\\
&\Rightarrow 3(x+1)\left(\frac{1}{3(x+1)}+2\right)=3(x+1)\left(\frac{2}{x+1}\right)\\
&\Leftrightarrow 1+6(x+1)=6\\
&\Leftrightarrow 1+6x+6=6\\
&\Leftrightarrow 6x+7=6\\
&\Leftrightarrow 6x=-1\\
&\Leftrightarrow \boxed{x=-\frac{1}{6}}
\end{aligned}$$

□

6. Solve the quadratic equation by completing the square.

$$x^2 - 6x = 2$$

Solution

$$\begin{aligned}
x^2 - 6x = 2 &\Leftrightarrow x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 2 + \left(\frac{-6}{2}\right)^2 \\
&\Leftrightarrow x^2 - 6x + 9 = 2 + 9 \\
&\Leftrightarrow (x-3)^2 = 11 \\
&\Leftrightarrow x - 3 = \pm\sqrt{11} \\
&\Leftrightarrow \boxed{x = 3 \pm \sqrt{11}}
\end{aligned}$$

$\square$

7. Solve the following equation using the quadratic formula.

$$x^2 + 4x - 6 = 0$$

Solution

$$\begin{aligned}
x^2 + 4x - 6 = 0 &\Leftrightarrow x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-6)}}{2(1)} \\
&\Leftrightarrow x = \frac{-4 \pm \sqrt{16 + 24}}{2} \\
&\Leftrightarrow x = \frac{-4 \pm \sqrt{40}}{2} \\
&\Leftrightarrow x = \frac{-4 \pm 2\sqrt{10}}{2} \\
&\Leftrightarrow x = \frac{\cancel{2}(-2 \pm \sqrt{10}}{\cancel{2}} \\
&\Leftrightarrow \boxed{x = -2 \pm \sqrt{10}}
\end{aligned}$$

$\square$

8. Compute the discriminant. Then determine the number and type of solutions of the given equation. (You do not need to find the solutions!)

$$3x^2 + 2x - 18 = 0$$

Solution

$$\begin{aligned} \text{Discriminant} &= (2)^2 - 4(3)(-18) \\ &= 4 + 216 \\ &= \boxed{220} \end{aligned}$$

The discriminant is positive so there are $\boxed{\text{two real solutions}}$

$\square$

9. Find all the roots.

$$2x^3 - 2x^2 - 18x + 18 = 0$$

Solution

$$\begin{aligned} 2x^3 - 2x^2 - 18x + 18 = 0 &\Leftrightarrow 2(x^3 - x^2 - 9x + 9) = 0 \\ &\Leftrightarrow x^3 - x^2 - 9x + 9 = 0 \\ &\Leftrightarrow x^2(x-1) - 9(x-1) = 0 \\ &\Leftrightarrow (x^2 - 9)(x-1) = 0 \\ &\Leftrightarrow (x-3)(x+3)(x-1) = 0 \\ &\Leftrightarrow \boxed{x = 3, -3, 1} \end{aligned}$$

$\square$

10. Solve the equation.

$$5x^{3/2} = 40$$

Solution

$$\begin{aligned} 5x^{3/2} = 40 &\Leftrightarrow x^{3/2} = 8 \\ &\Leftrightarrow (x^{3/2})^{2/3} = 8^{2/3} \\ &\Leftrightarrow x = 8^{2/3} \\ &\Leftrightarrow x = (\sqrt[3]{8})^2 \\ &\Leftrightarrow x = 2^2 \\ &\Leftrightarrow \boxed{x = 4} \end{aligned}$$

$\square$

11. Find the real solutions of the equation. Check your solutions!

$$\sqrt{10x-25} = x$$

Solution

$$\begin{aligned}\sqrt{10x-25} = x &\Rightarrow 10x-25 = x^2\\ &\Leftrightarrow x^2-10x+25=0\\ &\Leftrightarrow (x-5)^2=0\\ &\Leftrightarrow x-5=0\\ &\Leftrightarrow x=5\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{10(5)-25}=5 &\Leftrightarrow \sqrt{50-25}=5\\ &\Leftrightarrow \sqrt{25}=5\\ &\Leftrightarrow 5=5\checkmark\end{aligned}$$

Thus the solution is $\boxed{x=5}$

$\square$

12. Find the solutions(s) of the equation

$$|x+4| = 3$$

Solution

$$\begin{aligned}|x+4|=3 &\Leftrightarrow x+4=3 \text{ or } x+4=-3\\ &\Leftrightarrow \boxed{x=-1 \text{ or } x=-7}\end{aligned}$$

$\square$

13. Solve the absolute value inequality.

$$5 < |x+3|$$

Solution

$$\begin{aligned}5<|x+3| &\Leftrightarrow x+3>5 \text{ or } x+3<-5\\ &\Leftrightarrow \boxed{x>2 \text{ or } x<-8}\end{aligned}$$

$\square$

14. An architect is allowed 18 square yards of floor space to add a new bedroom onto a house. If the width must be 3 yards less than three times the length, then what is the width of the new bedroom.

Solution Let x be the length, then the width is $3x - 3$. Since the floor is 18 square yards we have

$$\begin{aligned} x(3x-3) = 18 &\Leftrightarrow 3x^2 - 3x = 18 \\ &\Leftrightarrow 3x^2 - 3x - 18 = 0 \\ &\Leftrightarrow 3(x^2 - x - 6) = 0 \\ &\Leftrightarrow 3(x-3)(x+2) = 0 \\ &\Leftrightarrow x = 3, -2 \end{aligned}$$

Since length must be positive, we have that the length is 3 yards. $3(3) - 3 = 9 - 3 = 6$ so the width is $\boxed{\text{6 yards}}$

□

15. Parts for an automobile repair cost \$76. The mechanic charges \$36 per hour. If you receive an estimate for at least \$112 and at most \$148 for fixing the car, what is the time interval that the mechanic will be working on the job?

Solution Let x be the amount of hours the mechanic works we have

$$\begin{aligned} 112 < 76 + 36x < 148 &\Leftrightarrow 36 < 36x < 72 \\ &\Leftrightarrow 1 < x < 2 \end{aligned}$$

Thus the mechanic will be working between $\boxed{\text{1 and two hours}}$

□

Bonus. Solve the following equation for x.

$$\left|\frac{px}{q} - a\right| = b,$$

where $b \geq 0$ and $p, q \neq 0$.

Solution

$$\begin{aligned} \left|\frac{px}{q} - a\right| = b &\Leftrightarrow \frac{px}{q} - a = b \text{ or } \frac{px}{q} - a = -b \\ &\Leftrightarrow \frac{px}{q} = a + b \text{ or } \frac{px}{q} = a - b \\ &\Leftrightarrow px = q(a+b) \text{ or } px = q(a-b) \\ &\Leftrightarrow \boxed{x = \frac{q(a+b)}{p} \text{ or } x = \frac{q(a-b)}{p}} \end{aligned}$$

□