Problem	Score	Worth
1		7
2		7
3		7
4		7
5		7
6		7
7		7
8		7
9		7
10		7
11		7
12		7
13		7
14		7
15		7
Total		100

There are 105 points available, you may earn up to 100.

Read the instructions carefully. Write legibly. Indicate your answers clearly. Show all your work. No notes, calculators, or other electronic devices allowed during the exam.
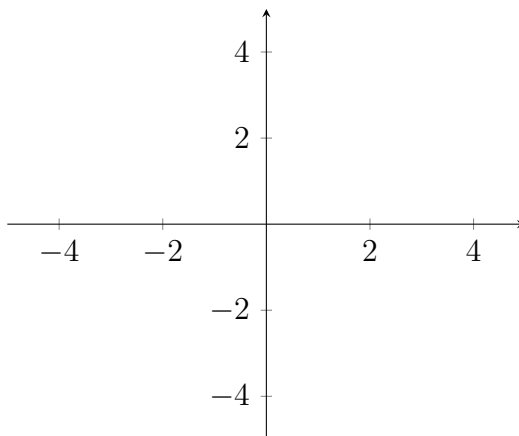
1. Determine if the following equation defines y as a function of x and explain why or why not.
$$x - 2 = y^2$$

No, consider (3,1) and (3,-1)

2. (a) Using transformations of graphs, graph the function
$$f(x) = -(x+1)^2 + 2$$

Start with the graph of $g(x) = x^2$. Shift it left 1 unit, to obtain the graph of $g(x+1) = (x+1)^2$. Reflect over the x-axis to obtain the graph of $-g(x+1) = -(x+1)^2$. Shift up 2 unit to obtain the graph of $-g(x+1) + 2 = -(x+1)^2 + 2 = f(x)$

(b) On what interval is $f(x)$ increasing? $(-\infty, -1)$

(c) On what interval is $f(x)$ decreasing? $(-1, \infty)$

3. Consider $f(x) = x^8 + x^4$.

Using the definition; determine if the function is odd, even or neither. Even

4. Consider $f(x) = 2x\sqrt{1 - x^2}$.

Using the definition; determine if the function is odd, even or neither. Odd

5. Evaluate the piecewise function at the given values of the independent variable.

$$f(x) = \begin{cases} x + 4, & \text{if } x \geq -4 \\ -(x + 4), & \text{if } x < -4 \end{cases}$$

(a) $f(0) = 4$

(b) $f(-7) = 3$

(c) $f(1) = 5$

6. Consider the line through the points (-5, -2) and (5,14).

(a) Find the slope of the line. $\frac{8}{5}$

(b) Find the point-slope equation of the line. $\frac{8}{5}(x + 5) = (y + 2)$

(c) Find the slope-intercept form of the equation of the line. $\frac{8}{5}x + 6 = y$

7. Consider the line passing through the point (3,-3) which is <u>perpendicular</u> to the line $y = \dfrac{1}{2}x + 1$.

  (a) Find the slope of the line. -2

  (b) Find the point-slope equation of the line. $y + 3 = -2(x - 3)$

  (c) Find the slope-intercept form of the equation of the line. $-2x + 3$

8. Using transformations of graphs, graph the function $f(x) = -\sqrt{x + 3}$.
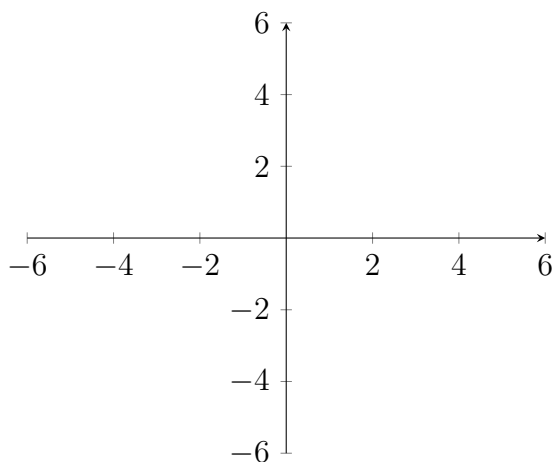
Start with the graph of $g(x) = \sqrt{x}$. Shift left 3, to obtain the graphs of $g(x + 3)$, then reflect over the x axis to obtain the graph of $-g(x + 3) = -\sqrt{x + 3}$

9. Let $f(x) = x - 8$ and let $g(x) = 4x^2$.

    (a)   i. Find $(f + g)(x)$ (simplify your answer) $4x^2 + x - 8$

           ii. What is the domain of $(f + g)(x)$ in interval notation? $(-\infty, \infty)$

    (b)   i. Find $(fg)(x)$ (simplify your answer) $4x^3 - 32x^2$

           ii. What is the domain of $(fg)(x)$ in interval notation? $(-\infty, \infty)$

    (c)   i. Find $\left(\dfrac{f}{g}\right)(x)$ (simplify your answer) $\frac{x-8}{4x^2}$

           ii. What is the domain of $\left(\dfrac{f}{g}\right)(x)$ in interval notation? $(-\infty, 0) \cup (0, \infty)$

10. Let $f(x) = -3x^2$ and let $g(x) = x + 6$.

    (a) Find $(f \circ g)(x)$ (simplify your answer) $-3x^2 - 36x - 108$

    (b) What is the domain of $(f \circ g)(x)$ in interval notation? $(-\infty, \infty)$

    (c) Find $(g \circ f)(x)$ (simplify your answer) $-3x^2 + 6$

    (d) What is the domain of $(g \circ f)(x)$ in interval notation? $(-\infty, \infty)$

11. The function $f(x) = \dfrac{9}{x} + 6$ is one-to-one.

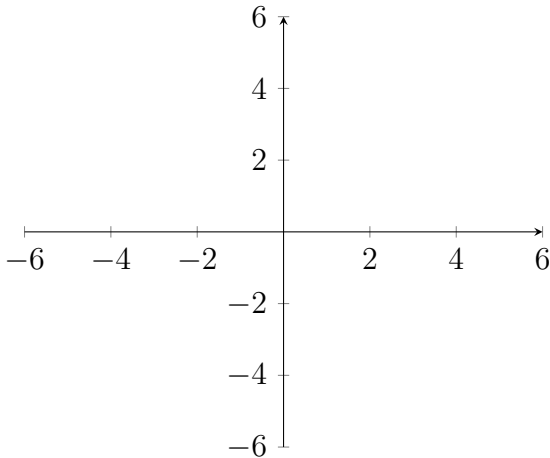   Find $f^{-1}(x)$. $f^{-1}(x) = \frac{9}{x-6})$

    Check!

    (a) $(f^{-1} \circ f)(x)$

    (b) $(f \circ f^{-1})(x)$

12. Let $f(x) = x^2 - 6, \ x \geq 0$.

    (a) Find $f^{-1}(x)$ $f^{-1}(x) = \sqrt{x+6}$

    (b) graph f and f^{-1}.

    (c) Using the graph determine:

        i. Domain of f = Range of f^{-1} = (use interval notation) $[0, \infty)$
        ii. Domain of f^{-1} = Range of f = (use interval notation) $[-6, \infty)$

13. Consider the general equation of a circle $x^2 + y^2 + 16x - 2y + 64 = 0$.

    (a) Write the equation of the circle in standard form. $(x+8)^2 + (y-1)^2 = 1$

    (b) What is the radius of the circle? 1

    (c) Give the point that represents the center of the circle. $(-8, 1)$

14. Consider the line segment connecting the points (1,4) and (-3,2).

    If this line segment is the diameter of a circle, what is the standard equation of the circle?
    $(x+1)^2 + (y-3)^2 = 5$

15. A person commutes to work a distance of 35 miles and returns on the same route at the end of the day. Their average rate on the return trip is 10 miler per hour faster than their average rate on the outgoing trip.

Create a function $T(x)$ where x represents the rate of the persons trip to work and the output of the function $T(x)$ is the total time spent on the outgoing and return trips in hours.

Hint: Time traveled $= \dfrac{\text{Distance traveled}}{\text{Rate of travel}}$.

$T(x) = \frac{35}{x} + \frac{35}{x+10} = \frac{70x+350}{x^2+10x}$