

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Solve the polynomial equations by factoring:

(a) $98x - 49 = 2x^3 - x^2$

(b) $18x - 9 = 2x^3 - x^2$

(c) $25y^3 - 3 = y - 75y^2$

(d) $x^3 + x^2 = 4x + 4$

Solution

(a)

$$\begin{aligned} 98x - 49 = 2x^3 - x^2 &\Leftrightarrow 2x^3 - x^2 - 98x + 49 = 0 \\ &\Leftrightarrow x^2(2x - 1) - 49(2x - 1) = 0 \\ &\Leftrightarrow (x^2 - 49)(2x - 1) = 0 \\ &\Leftrightarrow (x + 7)(x - 7)(2x - 1) = 0 \\ &\Leftrightarrow x + 7 = 0 \text{ or } x - 7 = 0 \text{ or } 2x - 1 = 0 \\ &\Leftrightarrow \boxed{x = -7 \text{ or } x = 7 \text{ or } x = \frac{1}{2}} \end{aligned}$$

(b)

$$\begin{aligned} 18x - 9 = 2x^3 - x^2 &\Leftrightarrow 2x^3 - x^2 - 18x + 9 = 0 \\ &\Leftrightarrow x^2(2x - 1) - 9(2x - 1) = 0 \\ &\Leftrightarrow (x^2 - 9)(2x - 1) = 0 \\ &\Leftrightarrow (x + 3)(x - 3)(2x - 1) = 0 \\ &\Leftrightarrow x + 3 = 0 \text{ or } x - 3 = 0 \text{ or } 2x - 1 = 0 \\ &\Leftrightarrow \boxed{x = -3 \text{ or } x = 3 \text{ or } x = \frac{1}{2}} \end{aligned}$$

(c)

$$\begin{aligned} 25y^3 - 3 = y - 75y^2 &\Leftrightarrow 25y^3 + 75y^2 - y - 3 = 0 \\ &\Leftrightarrow 25y^2(y + 3) - (y + 3) = 0 \\ &\Leftrightarrow (25y^2 - 1)(y + 3) = 0 \\ &\Leftrightarrow (5y - 1)(5y + 1)(y + 3) = 0 \\ &\Leftrightarrow 5y - 1 = 0 \text{ or } 5y + 1 = 0 \text{ or } y + 3 = 0 \\ &\Leftrightarrow \boxed{y = \frac{1}{5} \text{ or } y = -\frac{1}{5} \text{ or } y = -3} \end{aligned}$$

(d)

$$\begin{aligned}x^3 + x^2 = 4x + 4 &\Leftrightarrow x^3 + x^2 - 4x - 4 = 0 \\&\Leftrightarrow x^2(x + 1) - 4(x + 1) = 0 \\&\Leftrightarrow (x^2 - 4)(x + 1) = 0 \\&\Leftrightarrow (x - 2)(x + 2)(x + 1) = 0 \\&\Leftrightarrow x - 2 = 0 \text{ or } x + 2 = 0 \text{ or } x + 1 = 0 \\&\Leftrightarrow \boxed{x = 2 \text{ or } x = -2 \text{ or } x = -1}\end{aligned}$$

□

2. Solve the following radical equations and check all proposed solutions.

(a) $\sqrt{35 - 2x} = x$

(b) $\sqrt{2x + 19} - 8 = x$

(c) $\sqrt{x + 6} - 4 = x$

(d) $\sqrt{x + 8} - \sqrt{x - 4} = 2$

Solution

(a)

$$\begin{aligned}\sqrt{35 - 2x} = x &\rightarrow 35 - 2x = x^2 \\ &\Leftrightarrow x^2 + 2x - 35 = 0 \\ &\Leftrightarrow (x + 7)(x - 5) = 0 \\ &\Leftrightarrow x = -7, 5\end{aligned}$$

Checking, we have -7 isn't a valid solution so the answer is $x = 5$

(b)

$$\begin{aligned}\sqrt{2x + 19} - 8 = x &\Rightarrow 2x + 19 = 64 \\ &\Leftrightarrow 2x = 45 \\ &\Leftrightarrow x = \frac{45}{2}\end{aligned}$$

Checking, we have that this is a valid solution so the answer is $x = \frac{45}{2}$

(c)

$$\begin{aligned}\sqrt{x + 6} - 4 = x &\Leftrightarrow \sqrt{x + 6} = x + 4 \\ &\Rightarrow x + 6 = x^2 + 8x + 16 \\ &\Leftrightarrow x^2 + 7x + 10 = 0 \\ &\Leftrightarrow (x + 5)(x + 2) = 0 \\ &\Leftrightarrow x = -5, -2\end{aligned}$$

Checking, we have that -5 isn't a valid solution so the answer is $x = -2$

(d)

$$\begin{aligned}\sqrt{x + 8} - \sqrt{x - 4} = 2 &\Leftrightarrow \sqrt{x + 8} = 2 + \sqrt{x - 4} \\ &\Rightarrow x + 8 = 4 + 4\sqrt{x - 4} + x - 4 \\ &\Leftrightarrow \sqrt{x - 4} = 2 \\ &\Rightarrow x - 4 = 4 \\ &\Leftrightarrow x = 8\end{aligned}$$

Checking, we have that this is a valid solution so the answer is $x = 8$

□

3. Solve the following equations with rational exponents.

(a) $x^{3/2} = 125$

(b) $(x - 4)^{3/2} = 27$

(c) $6x^{7/4} - 30 = 0$

(d) $x^{3/2} = 64$

(e) $(x - 6)^{5/2} = 32$

Solution

(a)

$$\begin{aligned}x^{3/2} = 125 &\Leftrightarrow x = (125)^{2/3} \\ &\Leftrightarrow x = (\sqrt[3]{125})^2 \\ &\Leftrightarrow \boxed{x = 25}\end{aligned}$$

(b)

$$\begin{aligned}(x - 4)^{3/2} = 27 &\Leftrightarrow x - 4 = (27)^{2/3} \\ &\Leftrightarrow x - 4 = (\sqrt[3]{27})^2 \\ &\Leftrightarrow x - 4 = 9 \\ &\Leftrightarrow \boxed{x = 13}\end{aligned}$$

(c)

$$\begin{aligned}6x^{7/4} - 30 = 0 &\Leftrightarrow 6x^{7/4} = 30 \\ &\Leftrightarrow x^{7/4} = 5 \\ &\Leftrightarrow \boxed{x = 5^{4/7}}\end{aligned}$$

(d)

$$\begin{aligned}x^{3/2} = 64 &\Leftrightarrow x = (64)^{2/3} \\ &\Leftrightarrow x = (\sqrt[3]{64})^2 \\ &\Leftrightarrow \boxed{x = 16}\end{aligned}$$

(e)

$$\begin{aligned}(x - 6)^{5/2} = 32 &\Leftrightarrow x - 6 = (32)^{2/5} \\ &\Leftrightarrow x - 6 = (\sqrt[5]{32})^2 \\ &\Leftrightarrow x - 6 = 4 \\ &\Leftrightarrow \boxed{x = 10}\end{aligned}$$

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4. Use substitution to solve the following equations:

(a) $x - 8\sqrt{x} + 12 = 0$

(b) $4x^4 = 13x^2 - 9$

(c) $x^4 - 13x^2 + 36 = 0$

(d) $(x + 3)^2 + 12(x + 3) + 35 = 0$

(e) $x^{-2} - 5x^{-1} - 6 = 0$

Solution

(a) Let $u = \sqrt{x}$

$$\begin{aligned}x - 8\sqrt{x} + 12 = 0 &\Leftrightarrow u^2 - 8u + 12 = 0 \\&\Leftrightarrow (u - 6)(u - 2) = 0 \\&\Leftrightarrow u = 6, 2 \\&\Leftrightarrow \sqrt{x} = 6, 2 \\&\Leftrightarrow \boxed{x = 36, 4}\end{aligned}$$

(b) Let $u = x^2$

$$\begin{aligned}4x^4 = 13x^2 - 9 &\Leftrightarrow 2u^2 = 13u - 9 \\&\Leftrightarrow 4u^2 - 13u + 9 = 0 \\&\Leftrightarrow (4u - 9)(u - 1) = 0 \\&\Leftrightarrow u = \frac{9}{4}, 1 \\&\Leftrightarrow x^2 = \frac{9}{4}, 1 \\&\Leftrightarrow \boxed{x = \pm\frac{3}{2}, \pm 1}\end{aligned}$$

(c) Let $u = x^2$

$$\begin{aligned}x^4 - 13x^2 + 36 = 0 &\Leftrightarrow u^2 - 13u + 36 = 0 \\&\Leftrightarrow (u - 9)(u - 4) = 0 \\&\Leftrightarrow u = 9, 4 \\&\Leftrightarrow x^2 = 9, 4 \\&\Leftrightarrow \boxed{x = \pm 3, \pm 2}\end{aligned}$$

(d) Let $u = x + 3$

$$\begin{aligned}(x + 3)^2 + 12(x + 3) + 35 = 0 &\Leftrightarrow u^2 + 12u + 35 = 0 \\&\Leftrightarrow (u + 7)(u + 5) = 0 \\&\Leftrightarrow u = -7, -5 \\&\Leftrightarrow x + 3 = -7, -5 \\&\Leftrightarrow \boxed{x = -10, -8}\end{aligned}$$

(e) Let $u = x^{-1}$

$$\begin{aligned}x^{-2} - 5x^{-1} - 6 = 0 &\Leftrightarrow u^2 - 5u - 6 = 0 \\&\Leftrightarrow (u - 6)(u + 1) = 0 \\&\Leftrightarrow u = 6, -1 \\&\Leftrightarrow x^{-1} = 6, -1 \\&\Leftrightarrow \boxed{x = \frac{1}{6}, -1}\end{aligned}$$

□

5. Solve the following equations.

(a) $|x| = 13$

(b) $|x| = 10$

(c) $|x + 1| + 6 = 5$

(d) $|x + 4| + 8 = 6$

(e) $5|2x - 3| = 5$

Solution

(a)

$$|x| = 13 \Leftrightarrow \boxed{x = \pm 13}$$

(b)

$$|x| = 10 \Leftrightarrow \boxed{x = \pm 10}$$

(c)

$$\begin{aligned} |x + 1| + 6 = 5 &\Leftrightarrow |x + 1| = -1 \\ &\Rightarrow \boxed{\text{No Solution}} \end{aligned}$$

(d)

$$\begin{aligned} |x + 4| + 8 = 6 &\Leftrightarrow |x + 4| = -2 \\ &\Rightarrow \boxed{\text{No Solution}} \end{aligned}$$

(e)

$$\begin{aligned} 5|2x - 3| = 5 &\Leftrightarrow |2x - 3| = 1 \\ &\Leftrightarrow 2x - 3 = 1 \text{ or } 2x - 3 = -1 \\ &\Leftrightarrow 2x = 4 \text{ or } 2x = 2 \\ &\Leftrightarrow \boxed{x = 2 \text{ or } x = 1} \end{aligned}$$

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