Math 134 Spring 2018
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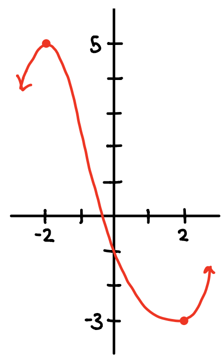
Classwork 14 Name: ________________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Graph a function f that has the given properties:

   (a) Domain $= (-\infty, \infty)$
   Increasing on $(-\infty, -2) \cup (2, \infty)$
   Decreasing on $(-2, 2)$
   Local maximum 5 at $x = -2$
   Local minimum -3 at $x = 2$

   Solution



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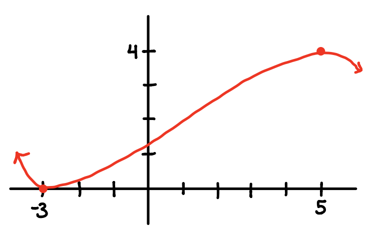
   (b) Domain $= (-\infty, \infty)$
   Decreasing on $(-\infty, -3) \cup (5, \infty)$
   Increasing on $(-3, 5)$
   Local maximum 4 at $x = 5$
   Local minimum 0 at $x = -3$

   Solution



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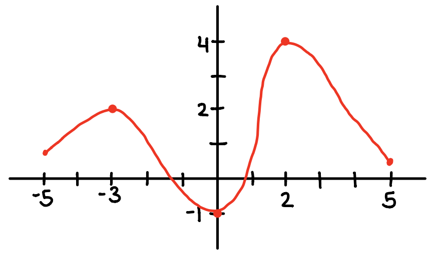
(c) Domain $= (-5, 5)$
Increasing on $(-5, -3) \cup (0, 2)$
Decreasing on $(-3, 0) \cup (2, 5)$
Local maxima 2 at $x = -3$ and 4 at $x = 2$.
Local minimum -1 at $x = 0$

Solution



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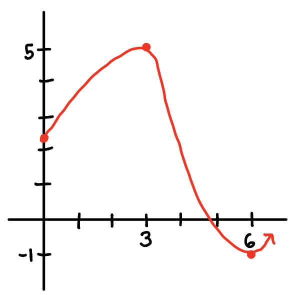
(d) Domain $= (0, \infty)$
Increasing on $(0, 3) \cup (6, \infty)$
Decreasing on $(3, 6)$
Local maximum 5 at $x = 3$
Local minimum -1 at $x = 6$

Solution



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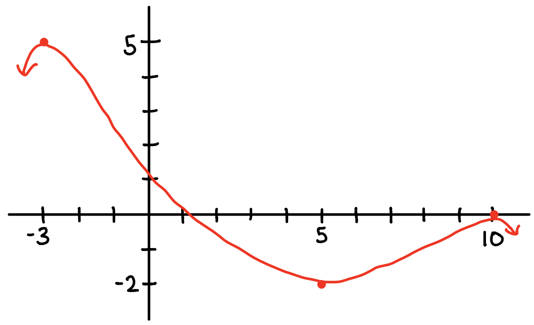
(e) Domain = $(-\infty, \infty)$
Increasing on $(-\infty, -3) \cup (5, 10)$
Decreasing on $(-3, 5) \cup (10, \infty)$
Local maxima 5 at $x = -3$ and 0 at $x = 10$
Local minimum -2 at $x = 5$

Solution



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2. Determine whether the given functions are even, odd, or neither.

(a) $x^4 - 7x^2 - 7$

(b) $3x^3 + x^2$

(c) $\dfrac{1+x^2}{x}$

(d) $x\sqrt{1-x^2}$

(e) $\dfrac{\sqrt{1+x^2}}{1+x}$

Solution

(a)

$$\begin{aligned} f(-x) &= (-x)^4 - 7(-x)^2 - 7 \\ &= x^4 - 7x^2 - 7 \\ &= f(x) \end{aligned}$$

$\boxed{\text{even}}$

(b)

$$\begin{aligned} f(-x) &= 3(-x)^3 + (-x)^2 \\ &= -3x^3 + x^2 \end{aligned}$$

$\boxed{\text{neither}}$

(c)

$$\begin{aligned} f(-x) &= \frac{1+(-x)^2}{-x} \\ &= \frac{1+x^2}{-x} \\ &= -\frac{1+x^2}{x} \\ &= -f(x) \end{aligned}$$

$\boxed{\text{odd}}$

(d)

$$\begin{aligned} f(-x) &= -x\sqrt{1-(-x)^2} \\ &= -x\sqrt{1-x^2} \\ &= -f(x) \end{aligned}$$

$\boxed{\text{odd}}$

(e)

$$f(-x) = \frac{\sqrt{1 + (-x)^2}}{1 + (-x)}$$

$$= \frac{\sqrt{1 + x^2}}{1 - x}$$

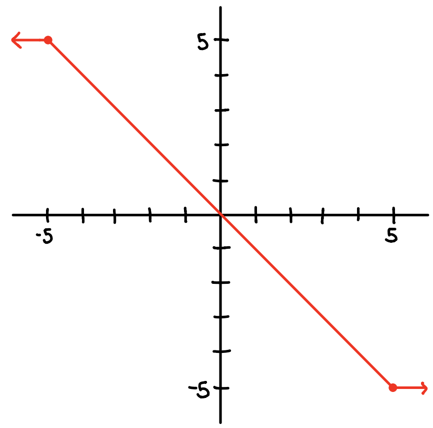
$\boxed{\text{neither}}$

$\square$

3. Graph the following piecewise functions.

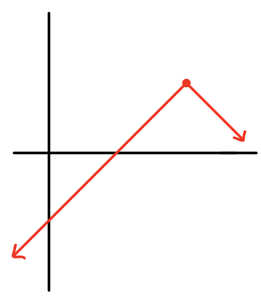
(a)

$$f(x) = \begin{cases} 5 & , -\infty < x \le -5 \\ -x & , -5 < x \le 5 \\ -5 & , 5 < x < \infty \end{cases}$$



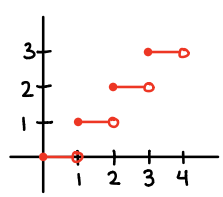
(b)

$$f(x) = \begin{cases} x - 2 & , -\infty < x \le 4 \\ 6 - x & , 4 < x < \infty \end{cases}$$



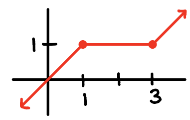
(c)

$$f(x) = \begin{cases} 0 & , 0 \le x < 1 \\ 1 & , 1 \le x < 2 \\ 2 & , 2 \le x < 3 \\ 3 & , 3 \le x < 4 \end{cases}$$



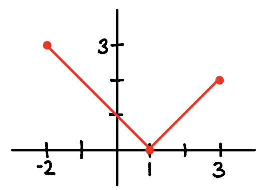
(d)

$$f(x) = \begin{cases} x & , -\infty < x < 1 \\ 1 & , 1 \leq x < 3 \\ x - 2 & , 3 \leq x < \infty \end{cases}$$



(e)

$$f(x) = \begin{cases} 1 - x & , -2 \leq x < 1 \\ x - 1 & , 1 \leq x \leq 3 \end{cases}$$



4. Given the function f, find and simplify the difference quotient.

(a) $f(x) = 3x^2 + 5$

(b) $f(x) = \dfrac{1}{x}$

(c) $f(x) = 9x - x^2$

(d) $f(x) = x^3$

(e) $f(x) = \sqrt{x}$

Solution

(a)

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{[3(x+h)^2 + 5] - [3x^2 + 5]}{h} \\
&= \frac{3(x^2 + 2xh + h^2) + 5 - 3x^2 - 5}{h} \\
&= \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{5} - \cancel{3x^2} - \cancel{5}}{h} \\
&= \frac{6xh + 3h^2}{h} \\
&= \frac{\cancel{h}(6x + 3h)}{\cancel{h}} \\
&= \boxed{6x + 3h}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)} \\
&= \frac{\frac{x\cancel{(x+h)}}{\cancel{x+h}} - \frac{\cancel{x}(x+h)}{\cancel{x}}}{hx(x+h)} \\
&= \frac{x - (x+h)}{hx(x+h)} \\
&= \frac{\cancel{x} - \cancel{x} - h}{hx(x+h)} \\
&= \frac{-\cancel{h}}{\cancel{h}x(x+h)} \\
&= \boxed{-\frac{1}{x(x+h)}}
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{[9(x+h)-(x+h)^2]-[9x-x^2]}{h} \\
&= \frac{9x+9h-(x^2+2xh+h^2)-9x+x^2}{h} \\
&= \frac{\cancel{9x}+9h-\cancel{x^2}-2xh-h^2-\cancel{9x}+\cancel{x^2}}{h} \\
&= \frac{9h-2xh-h^2}{h} \\
&= \frac{\cancel{h}(9-2x-h)}{\cancel{h}} \\
&= \boxed{9-2x-h}
\end{aligned}$$

(d)

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^3-x^3}{h} \\
&= \frac{(x+h)(x^2+2xh+h^2)-x^3}{h} \\
&= \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h} \\
&= \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}} \\
&= \boxed{3x^2+3xh+h^2}
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{x+h}-\sqrt{x}}{h} \\
&= \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \\
&= \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})} \\
&= \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} \\
&= \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} \\
&= \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}
\end{aligned}$$

□