Math 134 Spring 2018
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Classwork 17 Name: ______________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Starting with the graph of $y = x^2$, use transformations to graph the following function:

   (a) $f(x) = (x - 1)^2 + 3$
   (b) $f(x) = -(x + 1)^2 + 1$
   (c) $f(x) = (x + 3)^2 - 2$
   (d) $f(x) = -(x - 4)^2 - 1$
   (e) $f(x) = -(x + 3)^2 + 1$

   Solution

   (a) f is the graph of x^2 shifted right 1 unit and up 3 units.
   (b) f is the graph of x^2 reflected about the x-axis, shifted left 1 unit and up 1 unit.
   (c) f is the graph of x^2 shifted left 3 units and down 2 units.
   (d) f is the graph of x^2 reflected about the x-axis, shifted right 4 units and down 1 unit.
   (e) f is the graph of x^2 reflected about the x-axis, shifted left 3 units and up 1 unit.

$\square$

2. Starting with the graph of $y = |x|$, use transformations to graph the following function:

   (a) $g(x) = 2|x+3| - 2$
   (b) $g(x) = 3|x-2| - 2$
   (c) $g(x) = -2|x+1| + 5$
   (d) $g(x) = 3|x-3| + 1$
   (e) $g(x) = -2|x+1| + 2$

   Solution

   (a) g is the graph of $|x|$ vertically stretched by 2, shifted left 3 units, and down 2 units.
   (b) g is the graph of $|x|$ vertically stretched by 3, shifted right 2 units, and down 2 units.
   (c) g is the graph of $|x|$ reflected about the x-axis, vertically stretched by 2, shifted left 1 units and up 5 units.
   (d) g is the graph of $|x|$ vertically stretched by 3, shifted right 3 units, and up 1 unit.
   (e) g is the graph of $|x|$ reflected about the x-axis, vertically stretched by 2, shifted left 1 unit, and up 2 units.

$\square$

3. Starting with the graph of $y = \sqrt{x}$, use transformations to graph the following function:

   (a) $h(x) = -\sqrt{x-4}$

   (b) $h(x) = 2\sqrt{x+5}$

   (c) $h(x) = 3\sqrt{x-3}+1$

   (d) $h(x) = -2\sqrt{x+1}+1$

   (e) $h(x) = 4\sqrt{x}-5$

   Solution

   (a) h is the graph of $\sqrt{x}$ reflected about the x-axis and shifted right 4 units.

   (b) h is the graph of $\sqrt{x}$ vertically stretched by 2 and shifted left 5 units.

   (c) h is the graph of $\sqrt{x}$ vertically stretched by 3, shifted right 2 units, and up 1 unit.

   (d) h is the graph of $\sqrt{x}$ reflected about the x-axis, vertically stretched by 2, shifted left 1 unit, and up 1 unit.

   (e) h is the graph of $\sqrt{x}$ vertically stretched by 4 and shifted down 5 units.

$\square$

4. Starting with the graph of $y = \sqrt[3]{x}$, use transformations to graph the following function:

(a) $j(x) = \dfrac{1}{2}\sqrt[3]{x-1}$

(b) $j(x) = \dfrac{1}{2}\sqrt[3]{x+1}$

(c) $j(x) = \dfrac{1}{2}\sqrt[3]{x} + 2$

(d) $j(x) = \dfrac{1}{2}\sqrt[3]{x} - 3$

(e) $j(x) = \dfrac{1}{2}\sqrt[3]{x-2} + 1$

Solution

(a) j is the graph of $\sqrt[3]{x}$ vertically shrunk by $\dfrac{1}{2}$ and shifted right 1 unit.

(b) j is the graph of $\sqrt[3]{x}$ vertically shrunk by $\dfrac{1}{2}$ and shifted left 1 unit.

(c) j is the graph of $\sqrt[3]{x}$ vertically shrunk by $\dfrac{1}{2}$ and shifted up 2 units.

(d) j is the graph of $\sqrt[3]{x}$ vertically shrunk by $\dfrac{1}{2}$ and shifted down 3 units.

(e) j is the graph of $\sqrt[3]{x}$ vertically shrunk by $\dfrac{1}{2}$, shifted left 2 units, and up 1 unit.

$\square$

5. Starting with the graph of $y = x^3$, use transformations to graph the following function:

(a) $k(x) = -2(x-1)^3 + 1$

(b) $k(x) = \frac{1}{2}(x-2)^3 - 2$

(c) $k(x) = -\frac{1}{2}x^3 + 5$

(d) $k(x) = 2(x+2)^3 - 4$

(e) $k(x) = -\frac{1}{2}(x+3)^3 - 2$

Solution

(a) k is the graph of x^3 reflected about the x-axis, vertically stretched by 2, shifted right 1 unit, and up 1 unit.

(b) k is the graph of x^3 vertically shrunk by $\frac{1}{2}$, shifted left 2 units, and down 2 units.

(c) k is the graph of x^3 reflected about the x axis, vertically shrunk by $\frac{1}{2}$, and shifted up 5 units.

(d) k is the graph of x^3 vertically stretched by 2, shifted left 2 units, and down 4 units.

(e) k is the graph of x^3 reflected about the x-axis, vertically shrunk by $\frac{1}{2}$, shifted left 3 units, and down 2 units.

$\square$

6. Starting with the graph of $y = |x|$, use transformations to graph the following function:

   (a) $l(x) = |2x - 2|$
   (b) $l(x) = |3x + 3|$
   (c) $l(x) = \left|\frac{1}{2}x\right| + 5$
   (d) $l(x) = -|2x + 2|$
   (e) $l(x) = -2|3x + 3|$

   Solution

   (a)
   $$|2x - 2| = \left|2(x - 2)\right|$$
   l is the graph of $|x|$ horizontally shrunk by 2 and shifted right 2

   (b)
   $$|3x + 3| = \left|3(x + 1)\right|$$
   l is the graph of $|x|$ horizontally shrunk by 3 and shifted left 1

   (c) l is the graph of $|x|$ horizontally stretched by $\frac{1}{2}$ and shifted up 5 units

   (d)
   $$-|2x + 2| = -\left|2(x + 1)\right|$$
   l is the graph of $|x|$ reflected about the x-axis, horizontally shrunk by 2 and shifted left 1 unit.

   (e)
   $$-2|3x + 3| = -2\left|3(x + 1)\right|$$
   l is the graph of $|x|$ reflected about the x-axis, horizontally shrunk by 3, and shifted left 1 unit.

$\square$

7. Begin by completing the square. Then use transformations to graph the function:

   (a) $y = x^2 - 4x + 7$
   (b) $y = x^2 + 6x + 5$
   (c) $y = x^2 - 10x + 20$
   (d) $y = x^2 + 12x + 30$
   (e) $y = -x^2 + 4x + 3$

Solution

(a)

$$\begin{aligned} y &= (x^2 - 4x) + 7 \\ &= \left(x^2 - 4x + \left(\frac{-4}{2}\right)^2\right) + 7 - \left(\frac{-4}{2}\right)^2 \\ &= (x^2 - 4x + 4) + 7 - 4 \\ &= (x - 2)^2 + 3 \end{aligned}$$

Thus y is x^2 shifted right 2 units and up 3 units.

(b)

$$\begin{aligned} y &= (x^2 + 6x) + 5 \\ &= \left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) + 5 - \left(\frac{6}{2}\right)^2 \\ &= (x^2 + 6x + 9) + 5 - 9 \\ &= (x + 3)^2 - 4 \end{aligned}$$

Thus y is x^2 shifted left 3 units and down 4 units.

(c)

$$\begin{aligned} y &= (x^2 - 10x) + 20 \\ &= \left(x^2 - 10x + \left(\frac{-10}{2}\right)^2\right) + 20 - \left(\frac{-10}{2}\right)^2 \\ &= (x^2 - 10x + 25) + 20 - 25 \\ &= (x - 5)^2 - 5 \end{aligned}$$

Thus y is x^2 shifted right 5 units and down 5 units

(d)

$$\begin{aligned} y &= (x^2 + 12x) + 30 \\ &= \left(x^2 + 12x + \left(\frac{12}{2}\right)^2\right) + 30 - \left(\frac{12}{2}\right)^2 \\ &= (x^2 + 12x + 36) + 30 - 36 \\ &= (x + 6)^2 - 6 \end{aligned}$$

Thus y is x^2 shifted left 6 units and down 6 units

(e)

$$
\begin{aligned}
y &= -x^2 + 4x + 3 \\
&= -(x^2 - 4x) + 3 \\
&= -\left(x^2 - 4x + \left(\frac{-4}{2}\right)^2\right) + 3 + \left(\frac{-4}{2}\right)^2 \\
&= -(x^2 - 4x + 4) + 3 + 4 \\
&= -(x - 2)^2 + 7
\end{aligned}
$$

Thus y is x^2 reflected about the x-axis, shifted right 2 units, and up 7 units.

$\square$