Math 134 Spring 2018
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Classwork 18 Name: ________________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Find the domains of the following functions.

   (a) $f(x) = \dfrac{2}{x - 13}$

   (b) $f(x) = \dfrac{25}{x^2 - 3x - 88}$

   (c) $f(x) = \dfrac{1}{\frac{6}{x+5} - 2}$

   (d) $f(x) = \dfrac{1}{\sqrt{x + 7}}$

   (e) $f(x) = \dfrac{\sqrt{x - 4}}{x - 5}$

   Solution

   (a)

$$x - 13 \neq 0 \Leftrightarrow x \neq 13 \Rightarrow \boxed{(-\infty, 13) \cup (13, \infty)}$$

   (b)

$$x^2 - 3x - 88 \neq 0 \Leftrightarrow (x - 11)(x + 8) \neq 0 \Leftrightarrow x \neq 11, -8 \Rightarrow \boxed{(-\infty, -8) \cup (-8, 11) \cup (11, \infty)}$$

   (c) There are two restrictions $x + 5 \neq 0$ and $\dfrac{6}{x + 5} - 2 \neq 0$

$$x + 5 \neq 0 \Leftrightarrow \underline{x \neq -5}$$

$$\begin{aligned} \frac{6}{x+5} - 2 \neq 0 &\Leftrightarrow \frac{6}{x+5} \neq 2 \\ &\Leftrightarrow 6 \neq 2(x + 5) \\ &\Leftrightarrow 6 \neq 2x + 10 \\ &\Leftrightarrow 2x \neq -4 \\ &\Leftrightarrow \underline{x \neq -2} \end{aligned}$$

   Thus the domain is $\boxed{(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)}$

   (d) There are two restrictions that can be combined into one

$$\sqrt{x + 7} \neq 0 \text{ and } x + 7 \geq 0 \Leftrightarrow x + 7 > 0 \Leftrightarrow x > -7$$

   Thus the domain is $\boxed{(-7, \infty)}$

(e) There are two restrictions:

$$x - 4 \geq 0 \text{ and } x - 5 \neq 0 \Leftrightarrow x \geq 4 \text{ and } x \neq 5$$

Thus the domain is $\boxed{[4, 5) \cup (5, \infty)}$

$\square$

2. Find $f+g$, $f-g$, fg and $\frac{f}{g}$, then determine the domain for each function.

(a) $f(x) = 3x^2 + 1,\ g(x) = x + 3$

(b) $f(x) = 4x^2 + 16x,\ g(x) = x + 4$

(c) $f(x) = 4x^2 + 25x + 25,\ g(x) = x + 5$

(d) $f(x) = x^2 + 13x + 42,\ g(x) = x + 6$

(e) $f(x) = x^2 - 49,\ g(x) = x + 7$

Solution

(a)

$$\begin{aligned} f+g &= (3x^2+1) + (x+3) \\ &= \boxed{3x^2 + x + 4 \text{ Domain: } (-\infty, \infty)} \end{aligned}$$

$$\begin{aligned} f-g &= (3x^2+1) - (x+3) \\ &= 3x^2 + 1 - x - 3 \\ &= \boxed{3x^2 - x - 2 \text{ Domain: } (-\infty, \infty)} \end{aligned}$$

$$\begin{aligned} fg &= (3x^2+1)(x+3) \\ &= \boxed{2x^3 + 9x^2 + x + 3 \text{ Domain: } (-\infty, \infty)} \end{aligned}$$

$$\frac{f}{g} = \boxed{\frac{3x^2+1}{x+3} \text{ Domain: } x \neq -3 \Rightarrow (-\infty, -3) \cup (-3, \infty)}$$

(b)

$$f+g=(4x^2+16x)+(x+4)$$
$$=\boxed{4x^2+17x+4 \text{ Domain: } (-\infty,\infty)}$$

$$f-g=(4x^2+16x)-(x+4)$$
$$=4x^2+16x-x-4$$
$$=\boxed{4x^2+15x-4 \text{ Domain: } (-\infty,\infty)}$$

$$fg=(4x^2+16x)(x+4)$$
$$=4x^3+16x^2+16x^2+64x$$
$$=\boxed{4x^3+32x^2+64x \text{ Domain: } (-\infty,\infty)}$$

$$\frac{f}{g}=\frac{4x^2+16x}{x+4}$$
$$=\frac{4x\cancel{(x+4)}}{\cancel{x+4}}$$
$$=\boxed{4x, x\neq -4 \text{ Domain:}(-\infty,-4)\cup(-4,\infty)}$$

(c)

$$f+g=(4x^2+25x+25)+(x+5)$$
$$=4x^2+26x+30 \text{ Domain:}(-\infty,\infty)$$

$$f-g=(4x^2+25x+25)-(x+5)$$
$$=4x^2+25x+25-x-5$$
$$=\boxed{4x^2+24x+20 \text{ Domain: } (-\infty,\infty)}$$

$$fg=(4x^2+25x+25)(x+5)$$
$$=4x^3+20x^2+25x^2+125x+25x+125$$
$$=\boxed{4x^3+45x^2+150x+125 \text{ Domain: } (-\infty,\infty)}$$

$$\frac{f}{g}=\frac{4x^2+25x+25}{x+5}$$
$$=\frac{(4x+5)\cancel{(x+5)}}{\cancel{x+5}}$$
$$=\boxed{4x+5, x\neq -5 \text{ Domain: } (-\infty,-5)\cup(-5,\infty)}$$

(d)

$$\begin{aligned} f+g &= (x^2+13x+42)+(x+6) \\ &= \boxed{x^2+14x+48 \text{ Domain: } (-\infty,\infty)} \end{aligned}$$

$$\begin{aligned} f-g &= (x^2+13x+42)-(x+6) \\ &= x^2+13x+42-x-6 \\ &= \boxed{x^2+12x+36 \text{ Domain: } (-\infty,\infty)} \end{aligned}$$

$$\begin{aligned} fg &= (x^2+13x+42)(x+6) \\ &= x^3+6x^2+13x^2+78x+42x+252 \\ &= \boxed{x^3+19x^2+120x+252 \text{ Domain: } (-\infty,\infty)} \end{aligned}$$

$$\begin{aligned} \frac{f}{g} &= \frac{x^2+13x+42}{x+6} \\ &= \frac{(x+7)\cancel{(x+6)}}{\cancel{x+6}} \\ &= \boxed{x+7, x \neq -6 \text{ Domain: } (-\infty,-6)\cup(-6,\infty)} \end{aligned}$$

(e)

$$\begin{aligned} f+g &= (x^2-49)+(x+7) \\ &= \boxed{x^2+x-42 \text{ Domain: } (-\infty,\infty)} \end{aligned}$$

$$\begin{aligned} f-g &= (x^2-49)-(x+7) \\ &= x^2-49-x-7 \\ &= \boxed{x^2-x-56 \text{ Domain: } (-\infty,\infty)} \end{aligned}$$

$$\begin{aligned} fg &= (x^2-49)(x+7) \\ &= \boxed{x^3+7x^2-49x-343 \text{ Domain: } (-\infty,\infty)} \end{aligned}$$

$$\begin{aligned} \frac{f}{g} &= \frac{x^2-49}{x+7} \\ &= \frac{\cancel{(x+7)}(x-7)}{\cancel{x+7}} \\ &= \boxed{x-7, x \neq -7 \text{ Domain: } (-\infty,-7)\cup(-7,\infty)} \end{aligned}$$

□

3. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. What is $(f \circ g)(3)$ and $(g \circ f)(3)$?

   (a) $f(x) = 4x,\ g(x) = x + 5$

   (b) $f(x) = 2x,\ g(x) = x - 3$

   (c) $f(x) = 5x,\ g(x) = x + 2$

   (d) $f(x) = 7x,\ g(x) = x - 4$

   (e) $f(x) = 6x,\ g(x) = x + 1$

Solution

(a)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x+5) \\ &= 4(x+5) \\ &= \boxed{4x+20}\end{aligned}$$

$$\begin{aligned}(f \circ g)(3) &= 4(3) + 20 \\ &= 12 + 20 \\ &= \boxed{32}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(4x) \\ &= \boxed{4x+5}\end{aligned}$$

$$\begin{aligned}(g \circ f)(3) &= 4(3) + 5 \\ &= 12 + 5 \\ &= \boxed{17}\end{aligned}$$

(b)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x-3) \\ &= 2(x-3) \\ &\boxed{2x-6}\end{aligned}$$

$$\begin{aligned}(f \circ g)(3) &= 2(3) - 6 \\ &= 6 - 6 \\ &= \boxed{0}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(2x) \\ &= \boxed{2x-3}\end{aligned}$$

$$\begin{aligned}(g \circ f)(3) &= 2(3) - 3 \\ &= 6 - 3 \\ &= \boxed{3}\end{aligned}$$

(c)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x+2) \\ &= 5(x+2) \\ &= \boxed{5x+10}\end{aligned}$$

$$\begin{aligned}(f \circ g)(3) &= 5(3) + 10 \\ &= 15 + 10 \\ &= \boxed{25}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(5x) \\ &= \boxed{5x+2}\end{aligned}$$

$$\begin{aligned}(g \circ f)(3) &= 5(3) + 2 \\ &= 15 + 2 \\ &= \boxed{17}\end{aligned}$$

(d)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x-4) \\ &= 7(x-4) \\ &= \boxed{7x-24}\end{aligned}$$

$$\begin{aligned}(f \circ g)(3) &= 7(3)-24 \\ &= 21-24 \\ &= \boxed{-3}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(7x) \\ &= \boxed{7x-4}\end{aligned}$$

$$\begin{aligned}(g \circ f)(3) &= 7(3)-4 \\ &= 21-4 \\ &= \boxed{17}\end{aligned}$$

(e)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x+1) \\ &= 6(x+1) \\ &= \boxed{6x+6}\end{aligned}$$

$$\begin{aligned}(f \circ g)(3) &= 6(3)+6 \\ &= 18+6 \\ &= \boxed{24}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(6x) \\ &= \boxed{6x+1}\end{aligned}$$

$$\begin{aligned}(g \circ f)(3) &= 6(3)+1 \\ &= 18+1 \\ &= \boxed{19}\end{aligned}$$

$\square$

4. Find $(f \circ g)(x)$ and state the domain of this function.

(a) $f(x) = \dfrac{4}{x+3}$, $g(x) = \dfrac{1}{x}$

(b) $f(x) = \dfrac{3}{x+4}$, $g(x) = \dfrac{1}{x}$

(c) $f(x) = \dfrac{5}{x+7}$, $g(x) = \dfrac{1}{x}$

(d) $f(x) = \dfrac{1}{x+6}$, $g(x) = \dfrac{1}{x}$

(e) $f(x) = \dfrac{6}{x+2}$, $g(x) = \dfrac{1}{x}$

Solution

(a)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{1}{x}\right) \\ &= \frac{4}{\frac{1}{x}+3} \\ &= \boxed{\frac{4x}{1+3x}, x \neq 0}\end{aligned}$$

$$x \neq 0 \text{ and } \frac{1}{x} + 3 \neq 0 \Leftrightarrow x \neq -\frac{1}{3}$$

Thus the domain is

$$\boxed{\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 0\right) \cup (0, \infty)}$$

(b)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{1}{x}\right) \\ &= \frac{3}{\frac{1}{x}+4} \\ &= \boxed{\frac{3x}{1+4x}, x \neq 0}\end{aligned}$$

$$x \neq 0 \text{ and } \frac{1}{x} + 4 \neq 0 \Leftrightarrow x \neq -\frac{1}{4}$$

Thus the domain is

$$\boxed{\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, 0\right) \cup (0, \infty)}$$

(c)

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f\left(\frac{1}{x}\right) \\
&= \frac{5}{\frac{1}{x} + 7} \\
&= \boxed{\frac{5x}{1 + 7x}, x \neq 0}
\end{aligned}
$$

$$x \neq 0 \text{ and } \frac{1}{x} + 7 \neq 0 \Leftrightarrow x \neq -\frac{1}{7}$$

Thus the domain is

$$\boxed{\left(-\infty, -\frac{1}{7}\right) \cup \left(-\frac{1}{7}, 0\right) \cup (0, \infty)}$$

(d)

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f\left(\frac{1}{x}\right) \\
&= \frac{1}{\frac{1}{x} + 6} \\
&= \boxed{\frac{x}{1 + 6x}, x \neq 0}
\end{aligned}
$$

$$x \neq 0 \text{ and } \frac{1}{x} + 6 \neq 0 \Leftrightarrow x \neq -\frac{1}{6}$$

Thus the domain is

$$\boxed{\left(-\infty, -\frac{1}{6}\right) \cup \left(-\frac{1}{6}, 0\right) \cup (0, \infty)}$$

(e)

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f\left(\frac{1}{x}\right) \\
&= \frac{6}{\frac{1}{x} + 2} \\
&= \boxed{\frac{6x}{1+2x}, x \neq 0}
\end{aligned}
$$

$$x \neq 0 \text{ and } \frac{1}{x} + 2 \neq 0 \Leftrightarrow x \neq -\frac{1}{2}$$

Thus the domain is

$$\boxed{\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup (0, \infty)}$$

$\square$

5. Find two (non-identity) functions f and g such that $h(x) = (f \circ g)(x)$, where

(a) $h(x) = (2x - 1)^8$

(b) $h(x) = \sqrt[3]{3x - 4}$

(c) $h(x) = \sqrt{3x^3 - 4x + 7}$

(d) $h(x) = \dfrac{1}{2x - 3} + 1$

(e) $h(x) = |4x + 5|$

Solution

(a) $g(x) = 2x - 1,\ f(x) = x^8$

(b) $g(x) = 3x - 4,\ f(x) = \sqrt[3]{x}$

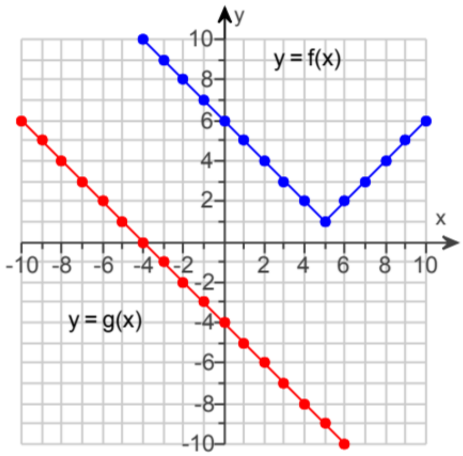
(c) $g(x) = 3x^3 - 4x + 7,\ g(x) = \sqrt{x}$

(d) $g(x) = 2x - 3,\ f(x) = \dfrac{1}{x} + 2$

(e) $g(x) = 4x + 5,\ f(x) = |x|$

$\square$

6. Consider the following graph of f and g.



(a) Find $(f \circ g)(0)$

(b) Find $(f \circ g)(-6)$

(c) Find $(g \circ f)(3)$

(d) Find $(g \circ f)(6)$

(e) Find $(f \circ f)(0)$

Solution

(a)
$$(f \circ g)(0) = f(g(0)) = f(-4) = \boxed{10}$$

(b)
$$(f \circ g)(-6) = f(g(-6)) = f(2) = \boxed{4}$$

(c)
$$(g \circ f)(3) = g(f(3)) = g(3) = \boxed{-7}$$

(d)
$$(g \circ f)(6) = g(f(6)) = g(2) = \boxed{-6}$$

(e)
$$(f \circ f)(0) = f(f(0)) = f(6) = \boxed{2}$$

□