

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Find the distance between the points.

- (a) (7, 9) and (19, 18)
- (b) (5, -3) and (-3, 6)
- (c) (0, 0) and (3, -4)
- (d) (-4, 3) and (3, -4)
- (e) (5, -2) and (10, 8)

**Solution**

(a)

$$\begin{aligned}\text{Distance} &= \sqrt{(19 - 7)^2 + (18 - 9)^2} \\ &= \sqrt{12^2 + 9^2} \\ &= \sqrt{144 + 81} \\ &= \sqrt{225} \\ &= \boxed{15}\end{aligned}$$

(b)

$$\begin{aligned}\text{Distance} &= \sqrt{(-3 - 5)^2 + (6 - (-3))^2} \\ &= \sqrt{(-8)^2 + 9^2} \\ &= \sqrt{64 + 81} \\ &= \boxed{\sqrt{145}}\end{aligned}$$

(c)

$$\begin{aligned}\text{Distance} &= \sqrt{(3 - 0)^2 + (-4 - 0)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= \boxed{5}\end{aligned}$$

(d)

$$\begin{aligned}\text{Distance} &= \sqrt{(3 - (-4))^2 + (-4 - 3)^2} \\ &= \sqrt{7^2 + (-7)^2} \\ &= \sqrt{49 + 49} \\ &= \sqrt{2 \cdot 49} \\ &= \boxed{7\sqrt{2}}\end{aligned}$$

(e)

$$\begin{aligned}\text{Distance} &= \sqrt{(10 - 5)^2 + (8 - (-2))^2} \\ &= \sqrt{5^2 + 10^2} \\ &= \sqrt{25 + 100} \\ &= \sqrt{125} \\ &= \boxed{5\sqrt{5}}\end{aligned}$$

□

2. Find the midpoint of the line segment with the given endpoints.

- (a) (8, 10) and (2, 6)
- (b) (-4, -7) and (-9, -8)
- (c) (2, 8) and (-5, -6)
- (d) (8, 10) and (2, 6)
- (e) (8, 4) and (2, -1)

**Solution**

(a)

$$\begin{aligned}\text{Midpoint} &= \left( \frac{8+2}{2}, \frac{10+6}{2} \right) \\ &= \left( \frac{10}{2}, \frac{16}{2} \right) \\ &= \boxed{(5, 8)}\end{aligned}$$

(b)

$$\begin{aligned}\text{Midpoint} &= \left( \frac{-4-9}{2}, \frac{-7-8}{2} \right) \\ &= \boxed{\left( \frac{-13}{2}, \frac{-15}{2} \right)} [5pt]\end{aligned}$$

(c)

$$\begin{aligned}\text{Midpoint} &= \left( \frac{2-5}{2}, \frac{8-6}{2} \right) \\ &= \left( \frac{-3}{2}, \frac{2}{2} \right) \\ &= \boxed{\left( -\frac{3}{2}, 1 \right)}\end{aligned}$$

(d) \*Same as part (a)\*

(e)

$$\begin{aligned}\text{Midpoint} &= \left( \frac{8+2}{2}, \frac{4-1}{2} \right) \\ &= \left( \frac{10}{2}, \frac{3}{2} \right) \\ &= \boxed{\left( 5, \frac{3}{2} \right)}\end{aligned}$$

□

3. Write the standard form of the equation of the circle with the given center and radius.

(a) Center:  $(4, 3)$ ,  $r = 5$

(b) Center:  $(-4, -3)$ ,  $r = 5$

(c) Center:  $(-2, 5)$ ,  $r = 6$

(d) Center:  $(3, -3)$ ,  $r = 7$

(e) Center:  $(0, -4)$ ,  $r = 8$

**Solution**

(a)  $(x - 4)^2 + (y - 3)^2 = 25$

(b)  $(x + 3)^2 + (y + 4)^2 = 25$

(c)  $(x + 2)^2 + (y - 5)^2 = 36$

(d)  $(x - 3)^2 + (y + 3)^2 = 42$

(e)  $x^2 + (y + 4)^2 = 64$

□

4. Identify the center and radius of the following equations.

(a)  $(x + 3)^2 + (y - 2)^2 = 12$

(b)  $(x - 6)^2 + (y + 1)^2 = 18$

(c)  $x^2 + (y + 72)^2 = 32$

(d)  $(x - 1)^2 + (y + 7)^2 = 75$

(e)  $(x + 10)^2 + (y + 5)^2 = 72$

**Solution**

(a) Center=  $(-3, 2)$ , Radius=  $\sqrt{12} = 2\sqrt{3}$

(b) Center=  $(6, -1)$ , Radius=  $\sqrt{18} = 3\sqrt{2}$

(c) Center=  $(0, -72)$ , Radius=  $\sqrt{32} = 4\sqrt{2}$

(d) Center=  $(1, -7)$ , Radius=  $\sqrt{75} = 5\sqrt{3}$

(e) Center=  $(-10, -5)$ , Radius=  $\sqrt{72} = 6\sqrt{2}$

□

5. Complete the square and write the equation of the circle in standard form. Then determine the center and radius of the circle and graph the equation.

(a)  $x^2 + y^2 + 10x + 4y + 4 = 0$

(b)  $x^2 + y^2 - 8x - 6y + 24 = 0$

(c)  $x^2 - 8x + y^2 + 7 = 0$

(d)  $x^2 + y^2 + 6x - 6y - 46 = 0$

(e)  $x^2 + y^2 - 4x - 10y + 20 = 0$

**Solution**

(a)

$$\begin{aligned} x^2 + y^2 + 10x + 4y + 4 = 0 &\Leftrightarrow (x^2 + 10x) + (y^2 + 4y) = -4 \\ &\Leftrightarrow \left(x^2 + 10x + \left(\frac{10}{2}\right)^2\right) + \left(y^2 + 4y + \left(\frac{4}{2}\right)^2\right) = -4 + \left(\frac{10}{2}\right)^2 + \left(\frac{4}{2}\right)^2 \\ &\Leftrightarrow (x^2 + 10x + 25) + (y^2 + 4y + 4) = 25 + 4 \\ &\Leftrightarrow \boxed{(x + 5)^2 + (y + 2)^2 = 29, C = (-5, -2) \text{ and } R = \sqrt{29}} \end{aligned}$$

(b)

$$\begin{aligned} x^2 + y^2 - 8x - 6y + 24 = 0 &\Leftrightarrow (x^2 - 8x) + (y^2 - 6y) = -24 \\ &\Leftrightarrow \left(x^2 - 8x + \left(\frac{-8}{2}\right)^2\right) + \left(y^2 - 6y + \left(\frac{-6}{2}\right)^2\right) = -24 + \left(\frac{-8}{2}\right)^2 + \left(\frac{-6}{2}\right)^2 \\ &\Leftrightarrow (x^2 - 8x + 16) + (y^2 - 6y + 9) = -24 + 16 + 9 \\ &\Leftrightarrow \boxed{(x - 4)^2 + (y - 3)^2 = 1, C = (4, 3) \text{ and } R = \sqrt{1} = 1} \end{aligned}$$

(c)

$$\begin{aligned} x^2 - 8x + y^2 + 7 = 0 &\Leftrightarrow (x^2 - 8x) + y^2 = -7 \\ &\Leftrightarrow \left(x^2 - 8x + \frac{-8}{2}\right) + y^2 = -7 + \left(\frac{-8}{2}\right)^2 \\ &\Leftrightarrow (x^2 - 8x + 16) + y^2 = -7 + 16 \\ &\Leftrightarrow \boxed{(x - 4)^2 + y^2 = 9, C = (4, 0) \text{ and } R = \sqrt{9} = 3} \end{aligned}$$

(d)

$$\begin{aligned}x^2 + y^2 + 6x - 6y - 46 = 0 &\Leftrightarrow (x^2 + 6x) + (y^2 - 6y) = 46 \\&\Leftrightarrow \left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) + \left(y^2 - 6y + \left(\frac{-6}{2}\right)^2\right) = 46 + \left(\frac{6}{2}\right)^2 + \left(\frac{-6}{2}\right)^2 \\&\Leftrightarrow (x^2 + 6x + 9) + (y^2 - 6y + 9) = 46 + 9 + 9 \\&\Leftrightarrow \boxed{(x + 3)^2 + (y - 3)^2 = 64, C = (-3, 3) \text{ and } R = \sqrt{64} = 8}\end{aligned}$$

(e)

$$\begin{aligned}x^2 + y^2 - 4x - 10y + 20 = 0 &\Leftrightarrow (x^2 - 4x) + (y^2 - 10y) = -20 \\&\Leftrightarrow \left(x^2 - 4x + \left(\frac{-4}{2}\right)^2\right) + \left(y^2 - 10y + \left(\frac{-10}{2}\right)^2\right) = -20 + \left(\frac{-4}{2}\right)^2 + \left(\frac{-10}{2}\right)^2 \\&\Leftrightarrow (x^2 - 4x + 4) + (y^2 - 10y + 25) = -20 + 4 + 25 \\&\Leftrightarrow \boxed{(x - 2)^2 + (y - 5)^2 = 9, C = (2, 5) \text{ and } R = \sqrt{9} = 3}\end{aligned}$$

□