Math 134 Spring 2018
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Classwork 21 Name: ______________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Find the coordinates of the vertex of the parabola defined by the quadratic functions.

   (a) $f(x) = -3(x+4)^2 + 7$ and $f(x) = x^2 - 10x + 25$
   (b) $f(x) = (x-3)^2 - 1$ and $f(x) = 4x^2 - 19x + 6$
   (c) $f(x) = (x-2)^2 + 3$ and $f(x) = -x^2 + 4x + 9$
   (d) $f(x) = 5(x+6)^2 - 3$ and $f(x) = -x^2 - 6x + 6$
   (e) $f(x) = -2(x-9)^2 + 27$ and $f(x) = 4x^2 - 16x + 4$

   Solution Note: You can also solve these problems by completing the square.

   (a) For $-3(x+4)^2 + 7$:

   $$\boxed{(-4, 7)}$$

   For $x^2 - 10x + 25$:

   $$\begin{aligned}
   \text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
   &= \left(\frac{10}{2}, f\left(\frac{10}{2}\right)\right) \\
   &= (5, f(5)) \\
   &= (5, 25 - 50 + 25) \\
   &= \boxed{(5, 0)}
   \end{aligned}$$

(b) For $(x-3)^2-1$:

$$\boxed{(3,-1)}$$

For $4x^2-19x+6$:

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
&= \left(\frac{19}{8}, f\left(\frac{19}{8}\right)\right) \\
&= \left(\frac{19}{8}, 4\left(\frac{19}{8}\right)^2 - 19\left(\frac{19}{8}\right) + 6\right) \\
&= \left(\frac{19}{8}, 4\cdot\frac{361}{64} - \frac{361}{8} + 6\right) \\
&= \left(\frac{19}{8}, \frac{361}{16} - \frac{361}{8} + 6\right) \\
&= \left(\frac{19}{8}, \frac{361}{16} - \frac{722}{16} + \frac{96}{16}\right) \\
&= \boxed{\left(\frac{19}{8}, -\frac{265}{16}\right)}
\end{aligned}$$

(c) For $(x-2)^2+3$:

$$\boxed{(2,3)}$$

For $-x^2+4x+9$:

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
&= \left(\frac{4}{-2}, f\left(\frac{4}{-2}\right)\right) \\
&= (-2, f(-2)) \\
&= (-2, -(-2)^2 + 4(-2) + 9) \\
&= (-2, -4 - 8 + 9) \\
&= \boxed{(-2,-3)}
\end{aligned}$$

(d) For $f(x) = 5(x+6)^2 - 3$:

$$\boxed{(-6,-3)}$$

For $-x^2 - 6x + 6$:

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
&= \left(\frac{6}{-2}, f\left(\frac{6}{-2}\right)\right) \\
&= (-3, f(-3)) \\
&= (-3, -(-3)^2 - 6(-3) + 6) \\
&= (-3, -9 + 18 + 6) \\
&= \boxed{(-3, 15)}
\end{aligned}$$

(e) For $-2(x-9)^2 + 27$:

$$\boxed{(9,27)}$$

For $4x^2 - 16x + 4$:

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
&= \left(\frac{16}{8}, f\left(\frac{16}{8}\right)\right) \\
&= (2, f(2)) \\
&= (2, 4(2)^2 - 16(2) + 4) \\
&= (2, 16 - 32 + 4) \\
&= \boxed{(2, -12)}
\end{aligned}$$

□

2. Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation for the parabola's axis of symmetry. Use the parabola to identify the function's domain and range.

   (a) $f(x) = 2(x+1)^2 - 3$
   (b) $f(x) = (x-3)^2 - 1$
   (c) $f(x) = 9 - (x-4)^2$
   (d) $f(x) = (x-2)^2 - 1$
   (e) $f(x) = 2(x-2)^2 - 2$

   Solution Use an online grapher to check your graphs.

   (a) Vertex: $(-1, -3)$
   Axis of symmetry: $x = -1$
   x-intercepts:

$$\begin{aligned} 0 = 2(x+1)^2 - 3 &\Leftrightarrow 2(x+1)^2 = 3 \\ &\Leftrightarrow (x+1)^2 = \frac{3}{2} \\ &\Leftrightarrow x + 1 = \pm\sqrt{\frac{3}{2}} \\ &\Leftrightarrow x = -1 \pm \sqrt{\frac{3}{2}} \end{aligned}$$

   y-intercept:

$$f(0) = 2(1)^2 - 3 = -1$$

   Domain: $(-\infty, \infty)$
   Range: $[-3, \infty)$

   (b) Vertex: $(3, -1)$
   Axis of symmetry: $x = 3$
   x-intercepts:

$$\begin{aligned} 0 = (x-3)^2 - 1 &\Leftrightarrow (x-3)^2 = 1 \\ &\Leftrightarrow x - 3 = \pm 1 \\ &\Leftrightarrow x = 3 \pm 1 \\ &\Leftrightarrow x = 4, 2 \end{aligned}$$

   y-intercept:

$$f(0) = (-3)^2 - 1 = 8$$

   Domain: $(-\infty, \infty)$
   Range: $[-1, \infty)$

(c)

$$f(x) = -(x-4)^2 + 9$$

Vertex: $(4, 9)$
Axis of symmetry: $x = 4$
x-intercepts:

$$\begin{aligned} 0 = 9 - (x-4)^2 &\Leftrightarrow (x-4)^2 = 9 \\ &\Leftrightarrow x - 4 = \pm 3 \\ &\Leftrightarrow x = 4 \pm 3 \\ &\Leftrightarrow x = 1, 7 \end{aligned}$$

y-intercept:

$$f(0) = 9 - (-4)^2 = -7$$

Domain: $(-\infty, \infty)$
Range: $(-\infty, 9]$

(d) Vertex: $(2, -1)$
Axis of symmetry: $x = 2$
x-intercepts:

$$\begin{aligned} 0 = (x-2)^2 - 1 &\Leftrightarrow (x-2)^2 = 1 \\ &\Leftrightarrow x - 2 = \pm 1 \\ &\Leftrightarrow x = 2 \pm 1 \\ &\Leftrightarrow = 3, 1 \end{aligned}$$

y-intercept:

$$f(0) = (-2)^2 - 1 = 3$$

Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$

(e) Vertex: $(2, -2)$
Axis of symmetry: $x = 2$
x-intercepts:

$$\begin{aligned} 0 = 2(x-2)^2 - 2 &\Leftrightarrow 2(x-2)^2 = 2 \\ &\Leftrightarrow (x-2)^2 = 1 \\ &\Leftrightarrow x - 2 = \pm 1 \\ &\Leftrightarrow x = 2 \pm 1 \\ &\Leftrightarrow x = 3, 1 \end{aligned}$$

Domain: $(-\infty, \infty)$
Range: $[-2, \infty)$

□

3. Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation for the parabola's axis of symmetry. Use the parabola to identify the function's domain and range.

   (a) $f(x) = -x^2 + 6x - 10$
   (b) $f(x) = 8x - x^2 - 17$
   (c) $f(x) = x^2 - 2x + 10$
   (d) $f(x) = 2x - x^2 - 2$
   (e) $f(x) = 3x^2 - 6x - 4$

Solution Use an online grapher to check your graphs.

(a)

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\
&= \left(\frac{-6}{-2}, f\left(\frac{-6}{-2}\right)\right)\\
&= (3, f(3))\\
&= (3, -9 + 18 - 10)\\
&= (3, -1)
\end{aligned}$$

Axis of symmetry: $x = 3$
x-intercepts:

$$\begin{aligned}
0 = -x^2 + 6x - 10 &\Leftrightarrow x^2 - 6x + 10 = 0\\
&\Leftrightarrow x = \frac{6 \pm \sqrt{36 - 4(1)(10)}}{2}\\
&\Leftrightarrow x = \frac{6 \pm \sqrt{-4}}{2}\\
&\Leftrightarrow x = \frac{6 \pm 2i}{2}\\
&\Leftrightarrow x = 3 \pm i
\end{aligned}$$

So no x-intercepts.

y-intercept:

$$f(0) = -10$$

Domain: $(-\infty, \infty)$
Range: $(-\infty, -1]$

(b)

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
&= \left(\frac{-8}{-2}, f\left(\frac{-8}{-2}\right)\right) \\
&= (4, f(4)) \\
&= (4, 32 - 16 - 17) \\
&= (4, -1)
\end{aligned}$$

Axis of symmetry: $x = 4$
x-intercepts:

$$\begin{aligned}
0 = 8x - x^2 - 17 &\Leftrightarrow x^2 - 8x + 17 = 0 \\
&\Leftrightarrow x = \frac{8 \pm \sqrt{64 - 4(17)}}{2} \\
&\Leftrightarrow x = \frac{8 \pm \sqrt{-4}}{2} \\
&\Leftrightarrow x = \frac{8 \pm 2i}{2} \\
&\Leftrightarrow x = 4 \pm i \\
&\text{So no } x\text{-intercepts.}
\end{aligned}$$

y-intercept:

$$f(0) = -17$$

Domain: $(-\infty, \infty)$
Range: $(-\infty, -1]$

(c)

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{2}{2}, f\left(\frac{2}{2}\right)\right)\\ &= (1, f(1))\\ &= (1, 1-2+10)\\ &= (1, 9)\end{aligned}$$

Axis of symmetry $x = 1$
x-intercepts:

$$\begin{aligned}0 = x^2 - 2x + 10 &\Leftrightarrow x = \frac{2 \pm \sqrt{4-4(10)}}{2}\\ &\Leftrightarrow x = \frac{2 \pm \sqrt{-36}}{2}\\ &\Leftrightarrow x = \frac{2 \pm 6i}{2}\\ &\Leftrightarrow x = 1 \pm 3i\end{aligned}$$

y-intercept:

$$f(0) = 10$$

Domain: $(-\infty, \infty)$
Range: $[9, \infty)$

(d)

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{-2}{-2}, f\left(\frac{-2}{2}\right)\right)\\ &= (1, f(1))\\ &= (1, 2-1-2)\\ &= (1, -1)\end{aligned}$$

Axis of symmetry: $x = 1$
x-intercepts:

$$\begin{aligned}0 = 2x - x^2 - 2 \Leftrightarrow & x^2 - 2x + 2 = 0\\ \Leftrightarrow & x = \frac{2 \pm \sqrt{4-4(2)}}{2}\\ \Leftrightarrow & x = \frac{2 \pm \sqrt{-4}}{2}\\ \Leftrightarrow & x = \frac{2 \pm 2i}{2}\\ \Leftrightarrow & x = 1 \pm i\end{aligned}$$

y-intercept:

$$f(0) = -2$$

Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$

(e)

$$\begin{aligned}
\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\
&= \left(\frac{6}{6}, f\left(\frac{6}{6}\right)\right) \\
&= (1, f(1)) \\
&= (1, 3 - 6 - 4) \\
&= (1, -7)
\end{aligned}$$

Axis of symmetry: $x = 1$
x-intercept:

$$\begin{aligned}
0 = 3x^2 - 6x - 4 &\Leftrightarrow x = \frac{6 \pm \sqrt{36 - 4(3)(-4)}}{6} \\
&\Leftrightarrow x = \frac{6 \pm \sqrt{36 + 48}}{6} \\
&\Leftrightarrow x = \frac{6 \pm \sqrt{84}}{6} \\
&\Leftrightarrow x = \frac{6 \pm 2\sqrt{21}}{6} \\
&\Leftrightarrow x = 1 \pm \frac{\sqrt{21}}{3}
\end{aligned}$$

y-intercept:

$$f(0) = -4$$

Domain: $(-\infty, \infty)$
Range: $[1, \infty)$

□

4. Determine, without graphing, whether the function has a minimum value or a maximum value. Find the value and where it occurs, and find the function's domain and range.

   (a) $f(x) = 2x^2 - 4x - 7$
   (b) $f(x) = 2x^2 - 12x - 1$
   (c) $f(x) = 3x^2 - 18x - 4$
   (d) $f(x) = -3x^2 + 12x - 1$
   (e) $f(x) = 2x^2 - 20x - 5$

   Solution

   (a) Minimum value

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{4}{4}, f\left(\frac{4}{4}\right)\right)\\ &= (1, f(1))\\ &= (1, 2-4-7)\\ &= (1, -9)\end{aligned}$$

   Thus the minimum value is -9 at $x = 1$.
   Domain: $(-\infty, \infty)$
   Range: $[-9.\infty)$

   (b) Minimum value

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{12}{4}, f\left(\frac{12}{4}\right)\right)\\ &= (3, f(3))\\ &= (3, 18-36-1)\\ &= (3, -19)\end{aligned}$$

   Thus the minimum value is -19 at $x = 3$.
   Domain: $(-\infty, \infty)$
   Range: $[-19, \infty)$

(c) Minimum value

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{18}{6} = f\left(\frac{18}{6}\right)\right)\\ &= (3, f(3))\\ &= (3, 27-54-4)\\ &= (3, -31)\end{aligned}$$

Thus the minimum value is -31 at $x = 3$.
Domain: $(-\infty, \infty)$
Range: $[-31, \infty)$

(d) Maximum value

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{-12}{-6}, f\left(\frac{-12}{-6}\right)\right)\\ &= (2, f(2))\\ &= (2, -12+24-1)\\ &= (2, 11)\end{aligned}$$

Thus the maximum value is 11 at $x = 2$.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 11]$

(e) Minimum value

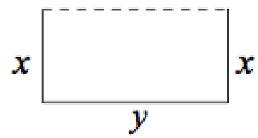
$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{20}{4}, f\left(\frac{20}{4}\right)\right)\\ &= (5, f(5))\\ &= (5, 50-100-5)\\ &= (5, -55)\end{aligned}$$

Thus the minimum value is -55 at $x = 5$.
Domain: $(-\infty, \infty)$
Range: $[-55, \infty)$

□

5. Joe has 850 meters of fencing and he wants to enclose a rectangular plot that borders a river. If Joe does not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

Solution The picture looks like the following:



The given equations we have are:

$$\text{Area} = xy \text{ and } 850 = 2x + y$$

Solving for y in the second equation gives

$$y = 850 = 2x$$

so the area is now

$$\text{Area} = x(850 - 2x) = -2x^2 + 850x$$

We want the maximum (vertex) of this parabola so we calculate:

$$\begin{aligned}\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) &= \left(\frac{-850}{-4}, f\left(\frac{-850}{-4}\right)\right)\\ &= \left(\frac{425}{2}, f\left(\frac{425}{2}\right)\right)\\ &= \left(\frac{425}{2}, -2\left(\frac{425}{2}\right)^2 + 850\left(\frac{425}{2}\right)\right)\\ &= \left(\frac{425}{2}, \frac{180625}{2}\right)\end{aligned}$$

Thus the maximum area is $\frac{180625}{2}$ m^2. The dimensions are

$$x = \frac{425}{2} \text{ m and } y = 850 - 425 = 425 \text{ m}$$

□