Math 134 Spring 2018
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Classwork 22 Name: _____

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Use the leading coefficient test to determine the end behavior of the graph of the given polynomial function.

   (a) $f(x) = 7x^5 + 2x^2 - 5x + 3$

   (b) $f(x) = 6x^4 + 5x^2 - x + 7$

   (c) $f(x) = -7x^4 + 4x^2 - 3x + 5$

   (d) $f(x) = -3x^5 + 5x^2 + x + 2$

   (e) $f(x) = -4x^4 + 3x^3 - x + 5$

   Solution

   (a) Falls to the left and rises to the right

   (b) Rises to the left and right

   (c) Falls to the left and right

   (d) Rises to the left and falls to the right

   (e) Falls to the left and right

□

2. Find the zeros for the polynomial function and give the multiplicity for each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

   (a) $f(x) = 6(x+8)(x-5)^2$

   (b) $f(x) = 6(x-4)(x-3)^3$

   (c) $f(x) = 9(x+3)(x-6)^2$

   (d) $f(x) = 4(x+5)(x-6)^3$

   (e) $f(x) = 2x^2(x-2)^3$

Solution

(a)

Zero	Multiplicity	Behavior
-8	1	Crosses
5	2	Touches and turns around

(b)

Zero	Multiplicity	Behavior
4	1	Crosses
3	3	Crosses

(c)

Zero	Multiplicity	Behavior
-3	1	Crosses
6	2	Touches and turns around

(d)

Zero	Multiplicity	Behavior
-5	1	Crosses
6	3	Crosses

(e)

Zero	Multiplicity	Behavior
0	2	Touches and turns around
2	3	Crosses

□

3. Use the intermediate value theorem to show that the polynomial has a real zero.

   (a) $f(x) = x^3 - 3x + 1$

   (b) $f(x) = x^3 + x + 1$

   (c) $f(x) = x^5 - 4x + 2$

   (d) $f(x) = x^5 + x - 3$

   (e) $f(x) = x^4 - 2x - 1$

   Solution

   (a)

   $$f(0) = 1 \text{ and } f(1) = -1$$

   f is a continuous function so by the intermediate value theorem there is a zero between 0 and 1.

   (b)

   $$f(-1) = -1 \text{ and } f(0) = 1$$

   f is a continuous function so by the intermediate value theorem there is a zero between -1 and 0.

   (c)

   $$f(0) = 2 \text{ and } f(1) = -1$$

   f is a continuous function so by the intermediate value theorem there is a zero between 0 and 1.

   (d)

   $$f(0) = -3 \text{ and } f(2) = 31$$

   f is a continuous function so by the intermediate value theorem there is a zero between 0 and 2.

   (e)

   $$f(-1) = 2 \text{ and } f(0) = -1$$

   f is a continuous function so by the intermediate value theorem there is a zero between -1 and 0.

□

4. For the given polynomial function , find the x-intercepts (as well as their multiplicities), the y-intercept, describe the end behavior, and sketch the graph.

   (a) $f(x) = -2x^4 + 6x^3$
   (b) $f(x) = x^4 + 10x^3 + 25x^2$
   (c) $f(x) = x^3 - 4x^2 - x + 4$
   (d) $f(x) = x^4 + 6x^3 + 9x^2$
   (e) $f(x) = -4x^4 + 8x^3$

   Solution Use a graphing utility to check your graphs.

   (a) x-intercepts:

$$\begin{aligned} 0 = -2x^4 + 6x^3 &\Leftrightarrow 2x^4 - 6x^3 = 0 \\ &\Leftrightarrow 2x^3(x-3) = 0 \\ &\Leftrightarrow x = 0 \text{ (multiplicity 3), } 3 \text{ (multiplicity 1)} \end{aligned}$$

   y-intercept:

$$f(0) = 0$$

   The graph falls to the left and right.

   (b) x-intercepts:

$$\begin{aligned} 0 = x^4 + 10x^3 + 25x^2 &\Leftrightarrow x^2(x^2 + 10x + 25) = 0 \\ &\Leftrightarrow x^2(x+5)^2 = 0 \\ &\Leftrightarrow x = 0 \text{ (multiplicity 2), } -5 \text{ (multiplicity 2)} \end{aligned}$$

   y-intercept:

$$f(0) = 0$$

   The graph rises to the left and right.

   (c) x-intercepts:

$$\begin{aligned} 0 = x^3 - 4x^2 - x + 4 &\Leftrightarrow x^2(x-4) - (x-4) = 0 \\ &\Leftrightarrow (x^2-1)(x-4) = 0 \\ &\Leftrightarrow (x-1)(x+1)(x-4) = 0 \\ &\Leftrightarrow x = 1 \text{ (multiplicity 1), } -1 \text{ (multiplicity 1), } 4 \text{ (multiplicity 1)} \end{aligned}$$

   y-intercept:

$$f(0) = 4$$

   The graph falls to the left and rises to the right.

   (d) x-intercepts:

$$\begin{aligned} 0 = x^4 + 6x^3 + 9x^2 &\Leftrightarrow x^2(x^2 + 6x + 9) = 0 \\ &\Leftrightarrow x^2(x+3)^2 = 0 \\ &\Leftrightarrow x = 0 \text{ (multiplicity 2), } -3 \text{ (multiplicity 2)} \end{aligned}$$

   y-intercept:

$$f(0) = 0$$

   The graph rises to the left and right.

(e) x-intercepts:

$$-4x^4 + 8x^3 = 0 \Leftrightarrow -4x^3(x-2) = 0$$
$$\Leftrightarrow x = 0 \text{ (multiplicity 3), } 2 \text{ (multiplicity 1)}$$

y-intercept:

$$f(0) = 0$$

The graph falls to the left and right.

□

5. Graph the polynomial function.

   (a) $f(x) = (x-2)^2(x+1)(x-3)$

   (b) $f(x) = (x-1)^2(x+4)(x-3)$

   (c) $f(x) = -x^2(x-1)(x+3)$

   (d) $f(x) = -x^2(x-2)(x+4)$

   (e) $f(x) = -2x^2(x-2)^2(x+5)$

   Solution Use a graphing utility to check your answers.

□