Math 134 Spring 2018
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Classwork 23 Name: ______________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Divide using long division. State the quotient, $q(x)$, and the remainder, $r(x)$.

   (a) $(8x^3 - 12x^2 - 14x - 15) \div (2x - 5)$
   (b) $(6x^3 - 2x^2 + 10x - 14) \div (2x - 2)$
   (c) $(9x^3 + 6x^2 - 9x - 6) \div (3x - 3)$
   (d) $(4x^3 + 4x^2 - 9x - 9) \div (2x - 3)$
   (e) $(4x^3 + 12x^2 + 7x - 7) \div (2x - 1)$

   Solution

   (a)
$$
\begin{array}{r|l}
 & 4x^2 \quad + 4x \quad + 3 \\
\hline
2x - 5 & 8x^3 - 12x^2 - 14x - 15 \\
 & -8x^3 + 20x^2 \\
\hline
 & \qquad 8x^2 - 14x \\
 & \qquad -8x^2 + 20x \\
\hline
 & \qquad\qquad 6x - 15 \\
 & \qquad\qquad -6x + 15 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

   Thus $\boxed{q(x) = 4x^2 + 4x + 3 \text{ and } r(x) = 0}$

   (b)
$$
\begin{array}{r|l}
 & 3x^2 \quad + 2x \quad + 7 \\
\hline
2x - 2 & 6x^3 - 2x^2 + 10x - 14 \\
 & -6x^3 + 6x^2 \\
\hline
 & \qquad 4x^2 + 10x \\
 & \qquad -4x^2 \quad + 4x \\
\hline
 & \qquad\qquad 14x - 14 \\
 & \qquad\qquad -14x + 14 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

   Thus $\boxed{q(x) = 3x^2 + 2x + 7 \text{ and } r(x) = 0}$

   (c)
$$
\begin{array}{r|l}
 & 3x^2 \quad + 5x + 2 \\
\hline
3x - 3 & 9x^3 \quad + 6x^2 \quad - 9x - 6 \\
 & -9x^3 \quad + 9x^2 \\
\hline
 & \qquad 15x^2 \quad - 9x \\
 & \qquad -15x^2 + 15x \\
\hline
 & \qquad\qquad 6x - 6 \\
 & \qquad\qquad -6x + 6 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

   Thus $\boxed{q(x) = 3x^2 + 5x + 2 \text{ and } r(x) = 0}$

(d)

$$
\begin{array}{r|rrrr}
 & & 2x^2 & +5x & +3 \\
\hline
2x-3 & 4x^3 & +4x^2 & -9x & -9 \\
 & -4x^3 & +6x^2 & & \\
\hline
 & & 10x^2 & -9x & \\
 & & -10x^2 & +15x & \\
\hline
 & & & 6x & -9 \\
 & & & -6x & +9 \\
\hline
 & & & & 0
\end{array}
$$

Thus $\boxed{q(x) = 2x^2 + 5x + 3 \text{ and } r(x) = 0}$

(e)

$$
\begin{array}{r|rrrr}
 & & 2x^2 & +7x & +7 \\
\hline
2x-1 & 4x^3 & +12x^2 & +7x & -7 \\
 & -4x^3 & +2x^2 & & \\
\hline
 & & 14x^2 & +7x & \\
 & & -14x^2 & +7x & \\
\hline
 & & & 14x & -7 \\
 & & & -14x & +7 \\
\hline
 & & & & 0
\end{array}
$$

Thus $\boxed{q(x) = 2x^2 + 7x + 7 \text{ and } r(x) = 0}$

$\square$

2. Divide using long division. State the quotient, $q(x)$, and the remainder, $r(x)$.

   (a) $(15x^4 + 18x^3 + x^2) \div (3x^2 + 2)$

   (b) $(18x^4 + 12x^3 + 17x^2) \div (2x^2 + 3)$

   (c) $(24x^4 + 9x^3 + 5x^2) \div (3x^2 + 1)$

   (d) $(4x^4 + 8x^3 + 4x^2) \div (2x^2 + 3)$

   (e) $(10x^4 + 10x^3 + 11x^2) \div (2x^2 + 3)$

Solution

(a)

$$
\begin{array}{rrrrrr}
 & & & 5x^2 & +6x & -3 \\ \cline{2-6}
3x^2+2 \big) & 15x^4 & +18x^3 & +x^2 & +0x & +0 \\
 & -15x^4 & & -10x^2 & & \\ \cline{2-4}
 & & 18x^3 & -9x^2 & +0x & \\
 & & -18x^3 & & -12x & \\ \cline{3-5}
 & & & -9x^2 & -12x & +0 \\
 & & & 9x^2 & & +6 \\ \cline{4-6}
 & & & & -12x & +6
\end{array}
$$

Thus $\boxed{q(x) = 5x^2 + 6x - 3 \text{ and } r(x) = -12x + 6}$

(b)

$$
\begin{array}{rrrrrr}
 & & & 9x^2 & +6x & -5 \\ \cline{2-6}
2x^2+3 \big) & 18x^4 & +12x^3 & +17x^2 & +0x & +0 \\
 & -18x^4 & & -27x^2 & & \\ \cline{2-4}
 & & 12x^3 & -10x^2 & +0x & \\
 & & -12x^3 & & -18x & \\ \cline{3-5}
 & & & -10x^2 & -18x & +0 \\
 & & & 10x^2 & & +15 \\ \cline{4-6}
 & & & & -18x & +15
\end{array}
$$

Thus $\boxed{q(x) = 9x^2 + 6x - 5 \text{ and } r(x) = -18x + 15}$

(c)

$$
\begin{array}{rrrrrr}
 & & & 8x^2 & +3x & -1 \\ \cline{2-6}
3x^2+1 \big) & 24x^4 & +9x^3 & +5x^2 & +0x & +0 \\
 & -24x^4 & & -8x^2 & & \\ \cline{2-4}
 & & 9x^3 & -3x^2 & +0x & \\
 & & -9x^3 & & -3x & \\ \cline{3-5}
 & & & -3x^2 & -3x & +0 \\
 & & & 3x^2 & & +1 \\ \cline{4-6}
 & & & & -3x & +1
\end{array}
$$

Thus $\boxed{q(x) = 8x^2 + 3x - 1 \text{ and } r(x) = -3x + 1}$

(d)

$$
\begin{array}{r|l}
 & 2x^2 + 4x - 1 \\
\hline
2x^2 + 3 & 4x^4 + 8x^3 + 4x^2 + 0x + 0 \\
 & -4x^4 \qquad - 6x^2 \\
\hline
 & 8x^3 - 2x^2 + 0x \\
 & -8x^3 \qquad - 12x \\
\hline
 & -2x^2 - 12x + 0 \\
 & 2x^2 \qquad + 3 \\
\hline
 & -12x + 3
\end{array}
$$

Thus $\boxed{q(x) = 2x^2 + 4x - 1 \text{ and } r(x) = -12x + 3}$

(e)

$$
\begin{array}{r|l}
 & 5x^2 + 5x - 2 \\
\hline
2x^2 + 3 & 10x^4 + 10x^3 + 11x^2 + 0x + 0 \\
 & -10x^4 \qquad - 15x^2 \\
\hline
 & 10x^3 - 4x^2 + 0x \\
 & -10x^3 \qquad - 15x \\
\hline
 & -4x^2 - 15x + 0 \\
 & 4x^2 \qquad + 6 \\
\hline
 & -15x + 6
\end{array}
$$

Thus $\boxed{q(x) = 5x^2 + 5x - 2 \text{ and } r(x) = -15x + 6}$

□

3. Divide using synthetic division.

(a) $(4x^5 - 2x^3 + 6x^2 - 7x + 6) \div (x - 2)$

(b) $(5x^5 - 4x^3 + 4x^2 - 3x + 7) \div (x - 2)$

(c) $(6x^5 - 4x^3 + 5x^2 - 4x + 2) \div (x - 2)$

(d) $(3x^5 - 5x^3 + 3x^2 - 4x + 3) \div (x - 2)$

(e) $(6x^5 - 5x^3 + 4x^2 - 3x + 5) \div (x - 2)$

Solution

(a)

$$\begin{array}{r|rrrrrr} & 4 & 0 & -2 & 6 & -7 & 6 \\ 2 & & 8 & 16 & 28 & 68 & 122 \\ \hline & 4 & 8 & 14 & 34 & 61 & 128 \end{array}$$

Thus $\boxed{q(x) = 4x^4 + 8x^3 + 14x^2 + 34x + 61 \text{ and } r(x) = 128}$

(b)

$$\begin{array}{r|rrrrrr} & 5 & 0 & -4 & 4 & -3 & 7 \\ 2 & & 10 & 20 & 32 & 72 & 138 \\ \hline & 5 & 10 & 16 & 36 & 69 & 145 \end{array}$$

Thus $\boxed{q(x) = 5x^4 + 10x^3 + 16x^2 + 36x + 69 \text{ and } r(x) = 145}$

(c)

$$\begin{array}{r|rrrrrr} & 6 & 0 & -4 & 5 & -4 & 2 \\ 2 & & 12 & 24 & 40 & 90 & 172 \\ \hline & 6 & 12 & 20 & 45 & 86 & 174 \end{array}$$

Thus $\boxed{q(x) = 6x^4 + 12x^3 + 20x^2 + 45x + 86 \text{ and } r(x) = 174}$

(d)

$$\begin{array}{r|rrrrrr} & 3 & 0 & -5 & 3 & -4 & 3 \\ 2 & & 6 & 12 & 14 & 34 & 60 \\ \hline & 3 & 6 & 7 & 17 & 30 & 63 \end{array}$$

Thus $\boxed{q(x) = 3x^4 + 6x^3 + 7x^2 + 17x + 30 \text{ and } r(x) = 63}$

(e)

$$\begin{array}{r|rrrrrr} & 6 & 0 & -5 & 4 & -3 & 5 \\ 2 & & 12 & 24 & 38 & 84 & 162 \\ \hline & 6 & 12 & 19 & 42 & 81 & 167 \end{array}$$

Thus $\boxed{q(x) = 6x^4 + 12x^3 + 19x^2 + 42x + 81 \text{ and } r(x) = 167}$

$\square$

4. Use synthetic division and the remainder theorem to find the indicated function value.

(a) $f(x) = x^4 + 4x^3 + 3x^2 - 2x - 6; f(4)$
(b) $f(x) = 6x^3 - 2x^2 - 3x + 1; f(-1)$
(c) $f(x) = 4x^3 - 3x^2 - 5x + 4; f(-1)$
(d) $f(x) = 3x^3 - 13x^2 + 3x - 3; f(3)$
(e) $f(x) = x^4 + 3x^3 + 5x^2 - 7x - 4; f(4)$

Solution

(a)

	1	4	3	−2	−6
4		4	32	140	552
	1	8	35	138	546

Thus $\boxed{f(4) = 546}$

(b)

	6	−2	−3	1
−1		−6	8	−5
	6	−8	5	−4

Thus $\boxed{f(-1) = -4}$

(c)

	4	−3	−5	4
−1		−4	7	−2
	4	−7	2	2

Thus $\boxed{f(-1) = 2}$

(d)

	3	−13	3	−3
3		9	−12	−27
	3	−4	−9	−30

Thus $\boxed{f(3) = -30}$

(e)

	1	3	5	−7	−4
4		4	28	132	500
	1	7	33	125	496

Thus $\boxed{f(4) = 496}$

□

5. Solve the equation $f(x) = 0$ given that a is a zero of $f(x)$.

(a) $f(x) = x^3 - 5x^2 + 2x + 8, a = -1$
(b) $f(x) = x^3 + 2x^2 - 5x - 6, a = 2$
(c) $f(x) = x^3 - 13x^2 + 47x - 35, a = 1$
(d) $f(x) = 9x^3 + 9x^2 - x - 1, a = -\frac{1}{3}$
(e) $f(x) = 16x^3 + 16x^2 - x - 1, a = -\frac{1}{4}$

Solution

(a)

$$\begin{array}{r|rrrr} & 1 & -5 & 2 & 8 \\ -1 & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

Thus we have

$$x^3 - 5x^2 + 2x + 8 = (x+1)(x^2 - 6x + 8) = (x+1)(x-4)(x-2)$$

So the solutions are

$$\boxed{x = -1, 4, 2}$$

(b)

$$\begin{array}{r|rrrr} & 1 & 2 & -5 & -6 \\ 2 & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

Thus we have

$$x^3 + 2x^2 - 5x - 6 = (x-2)(x^2 + 4x + 3) = (x-2)(x+3)(x+1)$$

So the solutions are

$$\boxed{x = 2, -3, -1}$$

(c)

$$\begin{array}{r|rrrr} & 1 & -13 & 47 & -35 \\ 1 & & 1 & -12 & 35 \\ \hline & 1 & -12 & 35 & 0 \end{array}$$

Thus we have

$$x^3 - 13x^2 + 47x - 35 = (x-1)(x^2 - 12x + 35) = (x-1)(x-5)(x-7)$$

So the solutions are

$$\boxed{x = 1, 5, 7}$$

(d)

$$\begin{array}{r|rrrr} & 9 & 9 & -1 & -1 \\ -\frac{1}{3} & & -3 & -2 & 1 \\ \hline & 9 & 6 & -3 & 0 \end{array}$$

Thus we have

$$\begin{aligned} 9x^3 + 9x^2 - x - 1 &= \left(x + \frac{1}{3}\right)(9x^2 + 6x - 3) \\ &= 3\left(x + \frac{1}{3}\right)(3x^2 + 2x - 1) \\ &= 3\left(x + \frac{1}{3}\right)(3x - 1)(x + 1) \end{aligned}$$

So the solutions are

$$\boxed{x = -\frac{1}{3}, \frac{1}{3}, -1}$$

(e)

$$\begin{array}{r|rrrr} & 16 & 16 & -1 & -1 \\ -\frac{1}{4} & & -4 & -3 & 1 \\ \hline & 16 & 12 & -4 & 0 \end{array}$$

Thus we have

$$\begin{aligned} 16x^3 + 16x^2 - x - 1 &= \left(x + \frac{1}{4}\right)(16x^2 + 12x - 4) \\ &= 4\left(x + \frac{1}{4}\right)(4x^2 + 3x - 1) \\ & 4\left(x + \frac{1}{4}\right)(4x - 1)(x + 1) \end{aligned}$$

So the solutions are

$$\boxed{x = -\frac{1}{4}, \frac{1}{4}, -1}$$

□