Math 134 Spring 2018
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Classwork 25 Name: ________________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

   (a) $f(x) = \dfrac{x}{x(x-4)}$

   (b) $f(x) = \dfrac{x^2-36}{x-6}$

   (c) $f(x) = \dfrac{x}{x(x-1)}$

   (d) $f(x) = \dfrac{x^2-16}{x+4}$

   (e) $f(x) = \dfrac{x(x-1)}{x^2-1}$

   Solution

   (a)
   $$f(x) = \frac{\cancel{x}}{\cancel{x}(x-4)} = \frac{1}{x-4}, x \neq 0$$
   Thus there is a $\boxed{\text{hole at } x = 0 \text{ and a vertical asymptote of } x = 4}$

   (b)
   $$f(x) = \frac{\cancel{(x-6)}(x+6)}{\cancel{x-6}} = x+6, x \neq 6$$
   Thus there is a $\boxed{\text{hole at } x = 6 \text{ and no vertical asymptote}}$

   (c)
   $$f(x) = \frac{\cancel{x}}{\cancel{x}(x-1)} = \frac{1}{x-1}x, \neq 0$$
   Thus there is a $\boxed{\text{hole at } x = 0 \text{ and a vertical asymptote of } x = 1}$

   (d)
   $$f(x) = \frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}} = x-4, x \neq -4$$
   Thus there is a $\boxed{\text{hole at } x = -4 \text{ and no vertical asymptote}}$

   (e)
   $$f(x) = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \frac{x}{x+1}, x \neq 1$$
   Thus there is a $\boxed{\text{hole at } x = 1 \text{ and a vertical asymptote of } x = -1}$

$\square$

2. Find the horizontal asymptote, if any, of the graph of the rational function.

(a) $f(x) = \dfrac{3x^2 - 5}{x(x-1)}$

(b) $f(x) = \dfrac{2x^2 - 1}{3x^2}$

(c) $f(x) = \dfrac{x^3 - 2x}{x^2 - 1}$

(d) $f(x) = \dfrac{x^2 - 9}{x^3 - 4x}$

(e) $f(x) = \dfrac{x^3 - 1}{2x^3 + 2}$

Solution

(a) $\boxed{y = \dfrac{3}{1} = 3}$

(b) $\boxed{y = \dfrac{2}{3}}$

(c) $\boxed{\text{None}}$

(d) $\boxed{y = 0}$

(e) $\boxed{y = \dfrac{1}{2}}$

$\square$

3. Graph the following rational function using transformations.

(a) $f(x) = -3 + \dfrac{1}{x}$

(b) $f(x) = \dfrac{1}{x-1} + 1$

(c) $f(x) = \dfrac{1}{x-2} - 1$

(d) $f(x) = 2 + \dfrac{1}{x}$

(e) $f(x) = \dfrac{1}{x-3} - 1$

Solution Use a graphing utility to check your answers.

(a) The graph of $\dfrac{1}{x}$ shifted down 3.

(b) The graph of $\dfrac{1}{x}$ shifted right 1 and up 1.

(c) The graph of $\dfrac{1}{x}$ shifted right 2 and down 1.

(d) The graph of $\dfrac{1}{x}$ shifted up 2.

(e) The graph of $\dfrac{1}{x}$ shifted right 3 and down 1.

$\square$

4. Graph the rational function. Identify the vertical and horizontal asymptotes, if , any. Plot the x and y intercepts.

   (a) $f(x) = \dfrac{2x^2 + 3x - 5}{2x^2 - 7x}$

   (b) $f(x) = \dfrac{3x^2 - x - 2}{2x^2 - 5x}$

   (c) $f(x) = \dfrac{2x^2 + x - 3}{2x^2 - 5x}$

   (d) $f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

   (e) $f(x) = \dfrac{3x^2 + 2x - 5}{2x^2 - 5x}$

Solution Use a graphing utility to check your answers.

(a)

$$f(x) = \frac{2x^2 + 3x - 5}{2x^2 - 7x} = \frac{(2x+5)(x-1)}{x(2x-7)}$$

So there are no holes, two vertical asymptotes of $x = 0$ and $x = \frac{7}{2}$, and a horizontal asymptote of $y = \frac{2}{2} = 1$.
$f(0)$ is undefined so there is no y-intercept.

$$f(x) = 0 \Leftrightarrow (2x+5)(x-1) = 0 \Leftrightarrow x = \frac{-5}{2}, 1$$

So there are two x-intercepts of $\frac{-5}{2}$ and 1.

(b)

$$f(x) = \frac{3x^2 - x - 2}{2x^2 - 5x} = \frac{(3x+2)(x-1)}{x(2x-5)}$$

So there are no holes, two vertical asymptotes of $x = 0$ and $x = \frac{5}{2}$, and a horizontal asymptote of $y = \frac{3}{2}$.
$f(0)$ is undefined so there is no y-intercept.

$$f(x) = 0 \Leftrightarrow (3x+2)(x-1) = 0 \Leftrightarrow x = -\frac{2}{3}, 1$$

So there are two x-intercepts of $\frac{-2}{3}, 1$.

(c)

$$f(x) = \frac{(2x-3)(x+1)}{x(2x-5)}$$

So there are no holes, two vertical asymptotes of $x = 0$ and $x = \frac{5}{2}$, and a horizontal asymptote of $y = \frac{2}{2} = 1$.

$f(0)$ is undefined so there is no y-intercept.

$$f(x) = 0 \Leftrightarrow (2x-3)(x+1) = 0 \Leftrightarrow x = \frac{3}{2}, -1$$

So there are two x-intercepts of $x = \frac{3}{2}, -1$.

(d)

$$f(x) = \frac{(3x+4)(x-1)}{x(2x-5)}$$

So there are no holes, two vertical asymptotes of $x = 0$ and $x = \frac{5}{2}$, and a horizontal asymptote of $y = \frac{3}{2}$.

$f(0)$ is undefined so there is no y-intercept.

$$f(x) = 0 \Leftrightarrow (3x+4)(x-1) = 0 \Leftrightarrow x = \frac{-4}{3}, 1$$

So there are two x-intercepts of $x = \frac{-4}{3}, 1$.

(e)

$$f(x) = \frac{(3x+5)(x-1)}{x(2x-5)}$$

So there are no holes, two vertical asymptotes of $x = 0$ and $x = \frac{5}{2}$, and a horizontal asymptote of $y = \frac{3}{2}$.

$f(0)$ is undefined so there is no y-intercept.

$$f(x) = 0 \Leftrightarrow (3x+5)(x-1) = 0 \Leftrightarrow x = \frac{-5}{3}, 1$$

So there are two x-intercepts of $x = \frac{-5}{3}, 1$

$\square$