Math 134 Spring 2018
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Classwork 2 Name: _____

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Evaluate each exponential expression.

   (a) $6^2 \cdot 2^4$

   (b) -3^3

   (c) $(-5)^2$

   (d) 6^0

   (e) 2^{-5}

   (f) $2^2 \cdot 2^5$

   (g) $7^2 \cdot 7^{-3}$

   (h) $\dfrac{3^4}{3^7}$

   Solution

   (a)

   $$6^2 \cdot 2^4 = 36 \cdot 16 = \boxed{576}$$

   (b)

   $$-3^3 = \boxed{-27}$$

   (c)

   $$(-5)^2 = \boxed{25}$$

   (d)

   $$6^0 = \boxed{1}$$

   (e)

   $$2^{-5} = \frac{1}{2^5} = \boxed{\frac{1}{32}}$$

   (f)

   $$\begin{aligned} 2^2 \cdot 2^5 &= 2^{2+5} \\ &= 2^7 \\ &= \boxed{128} \end{aligned}$$

   (g)

   $$\begin{aligned} 7^2 \cdot 7^{-3} &= 7^{2-3} \\ &= 7-1 \\ &= \boxed{\frac{1}{7}} \end{aligned}$$

   (h)

   $$\begin{aligned} \frac{3^4}{3^7} &= 3^{4-7} \\ &= 3^{-3} \\ &= \frac{1}{3^3} \\ &= \boxed{\frac{1}{27}} \end{aligned}$$

□

2. Simplify each exponential expression.

(a) $x^{-3}y$

(b) $x^3 \cdot x^2$

(c) $x^{-4}y$

(d) $\dfrac{x^{36}}{x^{29}}$

(e) $(3x)^3$

(f) $(8x^3)^2$

(g) $\left(-\dfrac{2}{x}\right)^2$

Solution

(a)

$$x^{-3}y = \boxed{\frac{y}{x^3}}$$

(b)

$$x^3 \cdot x^2 = x^{3+2} = \boxed{x^5}$$

(c)

$$x^{-4}y = \boxed{\frac{y}{x^4}}$$

(d)

$$\frac{x^{36}}{x^{29}} = x^{36-29} = \boxed{x^7}$$

(e)

$$(3x)^3 = (3)^3(x)^3 = \boxed{27x^3}$$

(f)

$$(8x^3)^2 = (8)^2(x^3)^2 = 64x^{3\cdot 2} = \boxed{64x^6}$$

(g)

$$\left(-\frac{2}{x}\right)^2 = \frac{(-2)^2}{(x)^2} = \boxed{\frac{4}{x^2}}$$

□

3. Simplify each exponential expression.

(a) $(-6x^3y)(-2x^5y^2)$

(b) $\left(\dfrac{x^{-2}y^8}{x^{-4}y^{12}}\right)^{-2}$

(c) $\left(\dfrac{-10a^{13}b^6}{30a^{18}b^{-3}}\right)^{2}$

(d) $\left(\dfrac{30x^{26}y^{45}}{41x^{-32}y}\right)^{0}$

Solution

(a)

$$\begin{aligned}(-6x^3y)(-2x^5y^2) &= (-6)(-2)x^3x^5yy^2\\ &= 12x^{3+5}y^{1+2}\\ &= \boxed{12x^8y^3}\end{aligned}$$

(b)

$$\begin{aligned}\left(\frac{x^{-2}y^8}{x^{-4}y^{12}}\right)^{-2} &= \left(\frac{x^{-2}}{x^{-4}}\cdot\frac{y^8}{y^{12}}\right)^{-2}\\ &= (x^{-2-(-4)}y^{8-12})^{-2}\\ &= (x^{-2+4}y^{-4})^{-2}\\ &= (x^2y^{-4})^{-2}\\ &= (x^2)^{-2}(y^{-4})^{-2}\\ &= x^{-4}y^8\\ &= \boxed{\frac{y^8}{x^4}}\end{aligned}$$

(c)

$$\left(\frac{-10a^{13}b^6}{30a^{18}b^{-3}}\right)^2 = \left(\frac{-10}{30}\cdot\frac{a^{13}}{a^{18}}\cdot\frac{b^6}{b^{-3}}\right)^2$$
$$= \left(-\frac{1}{3}a^{13-18}b^{6-(-3)}\right)^2$$
$$= \left(-\frac{1}{3}a^{-5}b^9\right)^2$$
$$= \left(-\frac{1}{3}\right)^2(a^{-5})^2(b^9)^2$$
$$= \frac{1}{9}a^{-10}b^{18}$$
$$= \boxed{\frac{b^{18}}{9a^{10}}}$$

(d)

$$\left(\frac{30x^{26}y^{45}}{41x^{-32}y}\right)^0 = \boxed{1}$$

□

4. Evaluate each expression, or indicate that the root is not a real number.

(a) $\sqrt{25}-\sqrt{4}$
(b) $\sqrt{25-4}$
(c) $\sqrt{(-6)^2}$
(d) $\sqrt{-25}$

Solution

(a)

$$\sqrt{25}-\sqrt{4} = 5-2$$
$$= \boxed{3}$$

(b)

$$\sqrt{25-4} = \boxed{\sqrt{21}}$$

(c)

$$\sqrt{(-6)^2} = \sqrt{36}$$
$$= \boxed{6}$$

(d) $\boxed{\text{Not a real number}}$

□

5. Use the product rule to simply each expression.

   (a) $\sqrt{45x^3}$

   (b) $\sqrt{3x^2} \cdot \sqrt{6x}$

   (c) $\sqrt{125y^2x} \cdot 10x^2$

Solution

(a)

$$\begin{aligned}\sqrt{45x^3} &= \sqrt{45}\sqrt{x^3} \\ &= \sqrt{9 \cdot 5}\sqrt{x^2 \cdot x} \\ &= \sqrt{9}\sqrt{5}\sqrt{x^2}\sqrt{x} \\ &= 3x\sqrt{5}\sqrt{x} \\ &= \boxed{3x\sqrt{5x}}\end{aligned}$$

(b)

$$\begin{aligned}\sqrt{3x^2} \cdot \sqrt{6x} &= x\sqrt{3} \cdot \sqrt{6x} \\ &= x\sqrt{18x} \\ &= x\sqrt{9 \cdot 2x} \\ &= x\sqrt{9}\sqrt{2x} \\ &= \boxed{3x\sqrt{2x}}\end{aligned}$$

(c)

$$\begin{aligned}\sqrt{125y^2x} \cdot 10x^2 &= 10x^2\sqrt{125y^2x} \\ &= 10x^2\sqrt{125}\sqrt{y^2}\sqrt{x} \\ &= 10x^2y\sqrt{25 \cdot 5}\sqrt{x} \\ &= 10x^2y\sqrt{25}\sqrt{5}\sqrt{x} \\ &= 10x^2y(5)\sqrt{5x} \\ &= \boxed{50x^2y\sqrt{5x}}\end{aligned}$$

□

6. Use the quotient rule to simplify the expressions. Assume $x > 0$.

(a) $\dfrac{\sqrt{3x^3}}{\sqrt{48x}}$

(b) $\dfrac{\sqrt{24x^4}}{\sqrt{4x^2}}$

(c) $\sqrt{\dfrac{121}{9}}$

Solution

(a)

$$\begin{aligned}\frac{\sqrt{3x^3}}{\sqrt{48x}} &= \sqrt{\frac{3x^3}{48x}} \\ &= \sqrt{\frac{\cancel{3x}(x^2)}{\cancel{3x}(16)}} \\ &= \sqrt{\frac{x^2}{16}} \\ &= \frac{x}{\sqrt{16}} \\ &= \boxed{\frac{x}{4}}\end{aligned}$$

(b)

$$\begin{aligned}\frac{\sqrt{24x^4}}{\sqrt{4x^2}} &= \sqrt{\frac{24x^4}{4x^2}} \\ &= \sqrt{\frac{\cancel{4x^2}(6x^2)}{\cancel{4x^2}}} \\ &= \sqrt{6x^2} \\ &= \sqrt{6}\sqrt{x^2} \\ &= \boxed{x\sqrt{6}}\end{aligned}$$

(c)

$$\begin{aligned}\sqrt{\frac{121}{9}} &= \frac{\sqrt{121}}{\sqrt{9}} \\ &= \boxed{\frac{11}{3}}\end{aligned}$$

□

7. Add or subtract terms whenever possible.

   (a) $6\sqrt{3} - 14\sqrt{3}$

   (b) $3\sqrt{5x} + 2\sqrt{5x} - 4\sqrt{5}$

   (c) $2\sqrt{54} - 3\sqrt{24} + \sqrt{96} - 5\sqrt{63}$

Solution

(a)

$$\begin{aligned} 6\sqrt{3} - 14\sqrt{3} &= (6 - 14)\sqrt{3} \\ &= \boxed{-8\sqrt{3}} \end{aligned}$$

(b)

$$\begin{aligned} 3\sqrt{5x} + 2\sqrt{5x} - 4\sqrt{5} &= (3 + 2)\sqrt{5x} - 4\sqrt{5} \\ &= \boxed{5\sqrt{5x} - 4\sqrt{5} \text{ or } \sqrt{5}(5\sqrt{x} - 4)} \end{aligned}$$

(c)

$$\begin{aligned} 2\sqrt{54} - 3\sqrt{24} + \sqrt{96} - 5\sqrt{63} &= 2\sqrt{9 \cdot 6} - 3\sqrt{4 \cdot 6} + \sqrt{16 \cdot 6} - 5\sqrt{9 \cdot 7} \\ &= 2\sqrt{9}\sqrt{6} - 3\sqrt{4}\sqrt{6} + \sqrt{16}\sqrt{6} - 5\sqrt{9}\sqrt{7} \\ &= 2(3)\sqrt{6} - 3(2)\sqrt{6} + 4\sqrt{6} - 5(3)\sqrt{7} \\ &= \cancel{6\sqrt{6}} - \cancel{6\sqrt{6}} + 4\sqrt{6} - 15\sqrt{7} \\ &= \boxed{4\sqrt{6} - 15\sqrt{7}} \end{aligned}$$

□

8. Rationalize the denominator.

(a) $\dfrac{1}{\sqrt{7}}$

(b) $\dfrac{\sqrt{3}}{\sqrt{5}}$

(c) $\dfrac{3}{3+\sqrt{5}}$

Solution

(a)

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$
$$= \boxed{\frac{\sqrt{7}}{7}}$$

(b)

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{\sqrt{3}\sqrt{5}}{5}$$
$$= \boxed{\frac{\sqrt{15}}{5}}$$

(c)

$$\frac{3}{3+\sqrt{5}} = \frac{3}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}}$$
$$= \frac{3(3-\sqrt{5})}{9-5}$$
$$= \boxed{\frac{3(3-\sqrt{5})}{4}}$$

□

9. Evaluate each expression.

   (a) $8^{1/3}$

   (b) $16^{-5/2}$

   (c) $125^{2/3}$

Solution

(a)

$$\begin{aligned} 8^{1/3} &= \sqrt[3]{8} \\ &= \boxed{2} \end{aligned}$$

(b)

$$\begin{aligned} 16^{-5/2} &= (16^{1/2})^{-5} \\ &= \sqrt{16}^{-5} \\ &= 4^{-5} \\ &= \frac{1}{4^5} \\ &= \boxed{\frac{1}{1024}} \end{aligned}$$

(c)

$$\begin{aligned} 125^{2/3} &= (125^{1/3})^2 \\ &= (\sqrt[3]{125})^2 \\ &= 5^2 \\ &= \boxed{25} \end{aligned}$$

□

10. Simplify the following expressions.

(a) $(7x^{1/3})(2x^{1/5})$

(b) $(y^{1/3})^6$

(c) $\dfrac{(2x^{1/4})^5}{x^{3/8}}$

(d) $\sqrt[3]{9} \cdot \sqrt[3]{6}$

(e) $\dfrac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}}$

(f) $\sqrt[3]{x^5}$

Solution

(a)

$$\begin{aligned}(7x^{1/3})(2x^{1/5}) &= 7(2)x^{1/3}x^{1/5} \\ &= 14x^{1/3+1/5} \\ &= 14x^{5/15+3/15} \\ &= \boxed{14x^{8/15}}\end{aligned}$$

(b)

$$\begin{aligned}(y^{1/3})^6 &= y^{1/3\cdot 6} \\ &= y^{6/3} \\ &= \boxed{y^2}\end{aligned}$$

(c)

$$\begin{aligned}\frac{(2x^{1/4})^5}{x^{3/8}} &= \frac{2^5 x^{1/4\cdot 5}}{x^{3/8}} \\ &= \frac{32x^{5/4}}{x^{3/8}} \\ &= 32x^{5/4-3/8} \\ &= 32x^{10/8-3/8} \\ &= \boxed{32x^{7/8}}\end{aligned}$$

(d)

$$\begin{aligned}\sqrt[3]{9} \cdot \sqrt[3]{6} &= \sqrt[3]{54} \\ &= \sqrt[3]{27 \cdot 2} \\ &= \sqrt[3]{27}\sqrt[3]{2} \\ &= \boxed{3\sqrt[3]{2}}\end{aligned}$$

(e)

$$\begin{aligned}\frac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} &= \frac{\sqrt[5]{32x^5 \cdot 2x}}{\sqrt[5]{2x}} \\ &= \frac{\sqrt[5]{32x^5}\cancel{\sqrt[5]{2x}}}{\cancel{\sqrt[5]{2x}}} \\ &= \sqrt[5]{32x^5} \\ &= \boxed{2x}\end{aligned}$$

(f)

$$\begin{aligned}\sqrt[3]{x^5} &= \sqrt[3]{x^3x^2} \\ &= \sqrt[3]{x^3}\sqrt[3]{x^2} \\ &= \boxed{x\sqrt[3]{x^2}}\end{aligned}$$

□