

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Determine which of the following are polynomials. If it is, write the polynomial in standard form and state its degree:

(a) $\frac{5x + 3}{x}$

(b) $x^2 + 7x^4 + 4x + 9x^3 - 4$

(c) $2x + 3x^{-1} - 5$

(d) $x^2 - x^3 + x^4 - 5$

Solution

(a)

(b)

(c)

(d)



2. Perform the indicated operation:

(a) $(-7x^3 + 2x^2 - 2x + 9) + (6x^3 + 6x^2 - 5x - 10)$

(b) $(3x^3 - 4x^2 + 3x - 5) - (6x^3 - 7x^2 - 10x + 4)$

(c) $(x + 3)(x^2 - 3x + 9)$

(d) $(3x - 7)(3x + 10)$

(e) $(2x + 3)(2x - 3)$

Solution

(a)

$$\begin{aligned}(-7x^3 + 2x^2 - 2x + 9) + (6x^3 + 6x^2 - 5x - 10) &= -7x^3 + 2x^2 - 2x + 9 + 6x^3 + 6x^2 - 5x - 10 \\ &= (-7x^3 + 6x^3) + (2x^2 + 6x^2) + (-2x - 5x) + (9 - 10) \\ &= (-7 + 6)x^3 + (2 + 6)x^2 + (-2 - 5)x - 1 \\ &= \boxed{-x^3 + 8x^2 - 7x - 1}\end{aligned}$$

(b)

$$\begin{aligned}(3x^3 - 4x^2 + 3x - 5) - (6x^3 - 7x^2 - 10x + 4) &= -3x^3 - 4x^2 + 3x - 5 - 6x^3 + 7x^2 + 10x - 4 \\ &= (-3x^3 - 6x^3) + (-4x^2 + 7x^2) + (3x + 10x) + (-5 - 4) \\ &= (-3 - 6)x^3 + (-4 + 7)x^2 + (3 + 10)x - 9 \\ &= \boxed{-9x^3 + 3x^2 + 13x - 9}\end{aligned}$$

(c)

$$\begin{aligned}(x + 3)(x^2 - 3x + 9) &= x(x^2 - 3x + 9) + 3(x^2 - 3x + 9) \\ &= x^3 - 3x^2 + 9x + 3x^2 - 9x + 27 \\ &= \boxed{x^3 + 27}\end{aligned}$$

(d)

$$\begin{aligned}(3x - 7)(3x + 10) &= 3x(3x + 10) - 7(3x + 10) \\ &= 9x^2 + 30x - 21x - 70 \\ &= \boxed{9x^2 + 9x - 70}\end{aligned}$$

(e)

$$\begin{aligned}(2x + 3)(2x - 3) &= 2x(2x - 3) + 3(2x - 3) \\ &= 4x^2 - 6x + 6x - 9 \\ &= \boxed{4x^2 - 9}\end{aligned}$$

□

3. Perform the indicated operation:

(a) $(5x + 6)^2$

(b) $(5x^2y - 8xy) + (10x^2y - 11xy)$

(c) $(x^3 - 5xy + 10y^2) - (7x^3 + 10xy + 9y^2)$

(d) $(x + 7y)(3x + 5y)$

Solution

(a)

$$\begin{aligned}(5x + 6)^2 &= (5x + 6)(5x + 6) \\ &= 5x(5x + 6) + 6(5x + 6) \\ &= 25x^2 + 30x + 30x + 36 \\ &= \boxed{25x^2 + 60x + 36}\end{aligned}$$

(b)

$$\begin{aligned}(5x^2y - 8xy) + (10x^2y - 11xy) &= 5x^2y - 8xy + 10x^2y - 11xy \\ &= (5x^2y + 10x^2y) + (-8xy - 11xy) \\ &= (5 + 10)x^2y + (-8 - 11)xy \\ &= \boxed{15x^2y - 19xy}\end{aligned}$$

(c)

$$\begin{aligned}(x^3 - 5xy + 10y^2) - (7x^3 + 10xy + 9y^2) &= x^3 - 5xy + 10y^2 - 7x^3 - 10xy - 9y^2 \\ &= (x^3 - 7x^3) + (-5xy - 10xy) + (10y^2 - 9y^2) \\ &= \boxed{-6x^3 - 15xy + y^2}\end{aligned}$$

(d)

$$\begin{aligned}(x + 7y)(3x + 5y) &= x(3x + 5y) + 7y(3x + 5y) \\ &= 3x^2 + 5xy + 21xy + 35y^2 \\ &= \boxed{3x^2 + 26xy + 35y^2}\end{aligned}$$

□

4. Factor the following polynomials completely or state the the polynomial is prime:

(a) $24x^2 + 18x$

(b) $x(x + 10) - 8(x + 10)$

(c) $x^3 - 4x^2 + 4x - 16$

(d) $3x^3 - 2x^2 - 21x + 14$

(e) $x^2 - 15x + 56$

Solution

(a)

$$\boxed{6x(4x + 3)}$$

(b)

$$\boxed{(x - 8)(x + 10)}$$

(c)

$$\begin{aligned}x^3 - 4x^2 + 4x - 16 &= (x^3 - 4x^2) + (4x - 16) \\ &= x^2(x - 4) + 4(x - 4) \\ &= \boxed{(x^2 + 4)(x - 4)}\end{aligned}$$

(d)

$$\begin{aligned}3x^3 - 2x^2 - 21x + 14 &= (3x^3 - 2x^2) + (-21x + 14) \\ &= x^2(3x - 2) - 7(3x - 2) \\ &= \boxed{(x^2 - 7)(3x - 2) \text{ or } (x - \sqrt{7})(x + \sqrt{7})(3x - 2)}\end{aligned}$$

(e)

$$\boxed{(x - 7)(x - 8)}$$

□

5. Factor the following polynomials completely or state the the polynomial is prime:

(a) $3a^2 - 8a - 28$

(b) $9x^2 - 4$

(c) $z^4 - 1$

(d) $y^2 + 10y + 25$

(e) $8x^3 + 1$

Solution

(a)

$$(3a - 14)(a + 2)$$

(b)

$$(3x - 2)(3x + 2)$$

(c)

$$(z^2 - 1)(z^2 + 1)$$

(d)

$$(y + 5)^2$$

(e)

$$(2x + 1)(4x^2 - 2x + 1)$$

□

6. Factor the following polynomials completely or state the the polynomial is prime:

(a) $6x^3 - 6x$

(b) $2x^2 + 24x + 64$

(c) $x^2 + 36$

(d) $48y^4 - 3y^2$

Solution

(a)

$$\begin{aligned} 6x^3 - 6x &= 6x(x^2 - 1) \\ &= \boxed{6x(x - 1)(x + 1)} \end{aligned}$$

(b)

$$\begin{aligned} 2x^2 + 24x + 64 &= 2(x^2 + 12x + 32) \\ &= \boxed{2(x + 8)(x + 4)} \end{aligned}$$

(c)

Prime

(d)

$$\begin{aligned} 48y^4 - 3y^2 &= 3y^2(16y^2 - 1) \\ &= \boxed{3y^2(4y - 1)(4y + 1)} \end{aligned}$$

□