

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Find all numbers for which the rational expression is undefined. if the rational expression is defined for all real numbers, state this:

(a) $\frac{13}{x+9}$

(b) $\frac{x-3}{x^2+4x-45}$

(c) $\frac{y+1}{y^2-1}$

(d) $\frac{x-1}{x^2+5x+4}$

Solution

(a)

$$x+9=0 \Leftrightarrow \boxed{x=-9}$$

(b)

$$\begin{aligned}x^2+4x-45=0 &\Leftrightarrow (x+9)(x-5)=0 \\&\Rightarrow x+9=0, x-5=0 \\&\Leftrightarrow \boxed{x=-9, x=5}\end{aligned}$$

(c)

$$\begin{aligned}y^2-1=0 &\Leftrightarrow (y-1)(y+1)=0 \\&\Rightarrow y-1=0, y+1=0 \\&\Leftrightarrow \boxed{y=1, y=-1}\end{aligned}$$

(d)

$$\begin{aligned}x^2+5x+4=0 &\Leftrightarrow (x+4)(x+1)=0 \\&\Rightarrow x+4=0, x+1=0 \\&\Leftrightarrow \boxed{x=-4, x=-1}\end{aligned}$$

□

2. Simplify the rational expressions. Find all numbers that must be excluded from the domain of the simplified rational expression in order for it to be equivalent to the original expression:

(a) $\frac{2x - 12}{x^2 - 12x + 36}$

(b) $\frac{y^2 + 9y + 8}{y^2 + y - 56}$

(c) $\frac{3x - 9}{x^2 - 6x + 9}$

(d) $\frac{x^2 - 14x + 49}{x^2 - 49}$

Solution

(a)

$$\begin{aligned}\frac{2x - 12}{x^2 - 12x + 36} &= \frac{2(\cancel{x-6})}{(\cancel{x-6})(x-6)} \\ &= \boxed{\frac{2}{x-6}, x \neq 6}\end{aligned}$$

(b)

$$\begin{aligned}\frac{y^2 + 9y + 8}{y^2 + y - 56} &= \frac{(\cancel{y+8})(y+1)}{(\cancel{y+8})(y-7)} \\ &= \boxed{\frac{y+1}{y-7}, y \neq -8, 7}\end{aligned}$$

(c)

$$\begin{aligned}\frac{3x - 9}{x^2 - 6x + 9} &= \frac{3(\cancel{x-3})}{(\cancel{x-3})(x-3)} \\ &= \boxed{\frac{3}{x-3}, x \neq 3}\end{aligned}$$

(d)

$$\begin{aligned}\frac{x^2 - 14x + 49}{x^2 - 49} &= \frac{(\cancel{x-7})(x-7)}{(\cancel{x-7})(x+7)} \\ &= \boxed{\frac{x-7}{x+7}, x \neq \pm 7}\end{aligned}$$

□

3. Perform the indicated operation:

$$(a) \frac{x^2 - 9}{x^2 - 6x + 9} \cdot \frac{6x - 18}{x + 3}$$

$$(b) \frac{x^2 - 9x + 18}{x^2 - 5x - 6} \cdot \frac{x^2 - 1}{x^2 - 9}$$

$$(c) \frac{y^2 - 1}{y} \div \frac{y + 1}{y - 1}$$

$$(d) \frac{x^2 - 64}{8x - 8} \div \frac{x^2 + 16x + 64}{x^2 + 7x - 8}$$

Solution

(a)

$$\begin{aligned} \frac{x^2 - 9}{x^2 - 6x + 9} \cdot \frac{6x - 18}{x + 3} &= \frac{\cancel{(x-3)}\cancel{(x+3)}}{\cancel{(x-3)}\cancel{(x-3)}} \cdot \frac{6\cancel{(x-3)}}{x+3} \\ &= \boxed{6, x \neq \pm 3} \end{aligned}$$

(b)

$$\begin{aligned} \frac{x^2 - 9x + 18}{x^2 - 5x - 6} \cdot \frac{x^2 - 1}{x^2 - 9} &= \frac{\cancel{(x-3)}\cancel{(x-6)}}{\cancel{(x-6)}\cancel{(x+1)}} \cdot \frac{(x-1)\cancel{(x+1)}}{\cancel{(x-3)}(x+3)} \\ &= \boxed{\frac{x-1}{x+3}, x \neq 6, -1, \pm 3} \end{aligned}$$

(c)

$$\begin{aligned} \frac{y^2 - 1}{y} \div \frac{y + 1}{y - 1} &= \frac{(y-1)\cancel{(y+1)}}{y} \cdot \frac{y-1}{\cancel{y+1}} \\ &= \boxed{\frac{(y-1)^2}{y}, y \neq 0, \pm 1} \end{aligned}$$

(d)

$$\begin{aligned} \frac{x^2 - 64}{8x - 8} \div \frac{x^2 + 16x + 64}{x^2 + 7x - 8} &= \frac{(x-8)(x+8)}{8(x-1)} \cdot \frac{x^2 + 7x - 8}{x^2 + 16x + 64} \\ &= \frac{(x-8)\cancel{(x+8)}}{8\cancel{(x-1)}} \cdot \frac{\cancel{(x+8)}\cancel{(x-1)}}{\cancel{(x+8)}(x+8)} \\ &= \boxed{\frac{x-8}{8}, x \neq 1, -8} \end{aligned}$$

□

4. Perform the indicated operation:

$$(a) \frac{7x+2}{6x+7} + \frac{5x+12}{6x+7}$$

$$(b) \frac{x^2-2x}{x^2+10x} + \frac{x^2+x}{x^2+10x}$$

$$(c) \frac{5}{x-2} + \frac{7}{x+3}$$

$$(d) \frac{15}{x^2+10x+25} + \frac{5}{x+5}$$

$$(e) \frac{4x}{x^2+2x-24} - \frac{3x}{x^2+x-20}$$

Solution

(a)

$$\begin{aligned} \frac{7x+2}{6x+7} + \frac{5x+12}{6x+7} &= \frac{(7x+2) + (5x+12)}{6x+7} \\ &= \frac{(7x+5x) + (2+12)}{6x+7} \\ &= \frac{12x+14}{6x+7} \\ &= \frac{2(6x+7)}{6x+7} \\ &= \boxed{2, x \neq -\frac{7}{6}} \end{aligned}$$

(b)

$$\begin{aligned} \frac{x^2-2x}{x^2+10x} + \frac{x^2+x}{x^2+10x} &= \frac{(x^2-2x) + (x^2+x)}{x^2+10x} \\ &= \frac{(x^2+x^2) + (-2x+x)}{x^2+10x} \\ &= \frac{2x^2-x}{x^2+10x} \\ &= \frac{x(2x-1)}{x(x+10)} \\ &= \boxed{\frac{2x-1}{x+10}, x \neq 0, -10} \end{aligned}$$

(c)

$$\begin{aligned}\frac{5}{x-2} + \frac{7}{x+3} &= \frac{5}{x-2} \cdot \frac{x+3}{x+3} + \frac{7}{x+3} \cdot \frac{x-2}{x-2} \\ &= \frac{5(x+3)}{(x-2)(x+3)} + \frac{7(x-2)}{(x+3)(x-2)} \\ &= \frac{5(x+3) + 7(x-2)}{(x+3)(x-2)} \\ &= \frac{5x + 15 + 7x - 14}{(x+3)(x-2)} \\ &= \boxed{\frac{12x + 1}{(x+3)(x-2)}, x \neq 2, -3}\end{aligned}$$

(d)

$$\begin{aligned}\frac{15}{x^2 + 10x + 25} + \frac{5}{x+5} &= \frac{15}{(x+5)(x+5)} + \frac{5}{x+5} \\ &= \frac{15}{(x+5)(x+5)} + \frac{5}{x+5} \cdot \frac{x+5}{x+5} \\ &= \frac{15}{(x+5)(x+5)} + \frac{5(x+5)}{(x+5)(x+5)} \\ &= \frac{15 + 5(x+5)}{(x+5)(x+5)} \\ &= \frac{15 + 5x + 25}{(x+5)(x+5)} \\ &= \boxed{\frac{5x + 40}{(x+5)^2}, x \neq -5}\end{aligned}$$

(e)

$$\begin{aligned}\frac{4x}{x^2 + 2x - 24} - \frac{3x}{x^2 + x - 20} &= \frac{4x}{(x+6)(x-4)} - \frac{3x}{(x+5)(x-4)} \\ &= \frac{4x}{(x+6)(x-4)} \cdot \frac{x+5}{x+5} - \frac{3x}{(x+5)(x-4)} \cdot \frac{x+6}{x+6} \\ &= \frac{4x(x+5)}{(x+6)(x-4)(x+5)} - \frac{3x(x+6)}{(x+5)(x-4)(x+6)} \\ &= \frac{(4x(x+5)) - (3x(x+6))}{(x+6)(x-4)(x+5)} \\ &= \frac{(4x^2 + 20x) - (3x^2 + 18x)}{(x+6)(x-4)(x+5)} \\ &= \frac{4x^2 + 20x - 3x^2 - 18x}{(x+6)(x-4)(x+5)} \\ &= \boxed{\frac{x^2 + 2x}{(x+6)(x-4)(x+5)}, x \neq -6, 4, -5}\end{aligned}$$

□

5. Simplify the following complex rational expressions:

$$(a) \frac{1 + \frac{1}{x}}{3 - \frac{1}{x}}$$

$$(b) \frac{\frac{x}{2} - 1}{x - 2}$$

$$(c) \frac{\frac{3}{x-3} - \frac{4}{x+3}}{\frac{11}{x^2-9}}$$

$$(d) \frac{\frac{6}{x^2+2x-15} - \frac{1}{x-3}}{\frac{1}{x+5} + 1}$$

Solution

(a)

$$\begin{aligned} \frac{1 + \frac{1}{x}}{3 - \frac{1}{x}} &= \frac{1 + \frac{1}{x}}{3 - \frac{1}{x}} \cdot \frac{x}{x} \\ &= \frac{\left(1 + \frac{1}{x}\right)x}{\left(3 - \frac{1}{x}\right)x} \\ &= \boxed{\frac{x+1}{3x-1}, x \neq 0, \frac{1}{3}} \end{aligned}$$

(b)

$$\begin{aligned} \frac{\frac{x}{2} - 1}{x - 2} &= \frac{\frac{x}{2} - 1}{x - 2} \cdot \frac{2}{2} \\ &= \frac{2\left(\frac{x}{2} - 1\right)}{x - 2} \\ &= \frac{x - 2}{2x - 4} \\ &= \frac{x - 2}{2(x - 2)} \\ &= \boxed{\frac{1}{2}, x \neq 2} \end{aligned}$$

(c)

$$\begin{aligned}\frac{\frac{3}{x-3} - \frac{4}{x+3}}{\frac{11}{x^2-9}} &= \frac{\frac{3}{x-3} - \frac{4}{x+3}}{\frac{11}{(x-3)(x+3)}} \\ &= \frac{\frac{3}{x-3} - \frac{4}{x+3}}{\frac{11}{(x-3)(x+3)}} \cdot \frac{(x-3)(x+3)}{(x-3)(x+3)} \\ &= \frac{(x-3)(x+3) \left(\frac{3}{x-3} - \frac{4}{x+3} \right)}{(x-3)(x+3) \left(\frac{11}{(x-3)(x+3)} \right)} \\ &= \frac{3(x+3) - 4(x-3)}{11} \\ &= \frac{3x+9-4x+12}{11} \\ &= \boxed{\frac{-x+21}{11}, x \neq \pm 3}\end{aligned}$$

(d)

$$\begin{aligned}\frac{\frac{6}{x^2+2x-15} - \frac{1}{x-3}}{\frac{1}{x+5} + 1} &= \frac{\frac{6}{(x+5)(x-3)} - \frac{1}{x-3}}{\frac{1}{x+5} + 1} \\ &= \frac{\frac{6}{(x+5)(x-3)} - \frac{1}{x-3}}{\frac{1}{x+5} + 1} \cdot \frac{(x+5)(x-3)}{(x+5)(x-3)} \\ &= \frac{(x+5)(x-3) \left(\frac{6}{(x+5)(x-3)} - \frac{1}{x-3} \right)}{(x+5)(x-3) \left(\frac{1}{x+5} + 1 \right)} \\ &= \frac{6 - (x+5)}{(x-3) + (x+5)(x-3)} \\ &= \frac{6-x-5}{x-3+x^2+2x-15} \\ &= \frac{-x+1}{x^2+3x-18} \\ &= \boxed{\frac{-x+1}{(x+6)(x-3)}, x \neq -5, 3, -6}\end{aligned}$$

□

6. **Challenge:** Perform the indicated operations. Simplify if possible:

$$(a) \left(\frac{2x+3}{x+1} \cdot \frac{x^2+4x-5}{2x^2+x-3} \right) - \frac{2}{x+2}$$

$$(b) \frac{1}{x^2-2x-8} \div \left(\frac{1}{x-4} - \frac{1}{x+2} \right)$$

$$(c) \left(2 - \frac{6}{x+1} \right) \left(1 + \frac{3}{x-2} \right)$$

$$(d) \left(4 - \frac{3}{x+2} \right) \left(1 + \frac{5}{x-1} \right)$$

Solution

(a)

$$\begin{aligned} \left(\frac{2x+3}{x+1} \cdot \frac{x^2+4x-5}{2x^2+x-3} \right) - \frac{2}{x+2} &= \left(\frac{\cancel{2x+3} \cdot (x+5)\cancel{(x-1)}}{(x+1)\cancel{(2x+3)}\cancel{(x-1)}} \right) - \frac{2}{x+2} \\ &= \frac{x+5}{x+1} - \frac{2}{x+2} \\ &= \frac{x+5}{x+1} \cdot \frac{x+2}{x+2} - \frac{2}{x+2} \cdot \frac{x+1}{x+1} \\ &= \frac{(x+5)(x+2)}{(x+1)(x+2)} - \frac{2(x+1)}{(x+2)(x+1)} \\ &= \frac{(x+5)(x+2) - 2(x+1)}{(x+1)(x+2)} \\ &= \frac{(x^2+7x+10) - (2x+2)}{(x+1)(x+2)} \\ &= \frac{x^2+7x+10-2x-2}{(x+1)(x+2)} \\ &= \boxed{\frac{x^2+5x+8}{(x+1)(x+2)}, x \neq -1, 1, -\frac{3}{2}} \end{aligned}$$

(b)

$$\begin{aligned}\frac{1}{x^2 - 2x - 8} \div \left(\frac{1}{x-4} - \frac{1}{x+2} \right) &= \frac{1}{(x-4)(x+2)} \div \left(\frac{1}{x-4} \cdot \frac{x+2}{x+2} - \frac{1}{x+2} \cdot \frac{x-4}{x-4} \right) \\ &= \frac{1}{(x-4)(x+2)} \div \left(\frac{x+2}{(x-4)(x+2)} - \frac{x-4}{(x+2)(x-4)} \right) \\ &= \frac{1}{(x-4)(x+2)} \div \left(\frac{(x+2) - (x-4)}{(x-4)(x+2)} \right) \\ &= \frac{1}{(x-4)(x+2)} \div \frac{x+2-x+4}{(x-4)(x+2)} \\ &= \frac{1}{(x-4)(x+2)} \div \frac{6}{(x-4)(x+2)} \\ &= \frac{1}{\cancel{(x-4)(x+2)}} \cdot \frac{\cancel{(x-4)(x+2)}}{6} \\ &= \boxed{\frac{1}{6}, x \neq 4, -2}\end{aligned}$$

(c)

$$\begin{aligned}\left(2 - \frac{6}{x+1} \right) \left(1 + \frac{3}{x-2} \right) &= \left(2 \cdot \frac{x+1}{x+1} - \frac{6}{x+1} \right) \left(1 \cdot \frac{x-2}{x-2} + \frac{3}{x-2} \right) \\ &= \left(\frac{2(x+1) - 6}{x+1} \right) \left(\frac{x-2+3}{x-2} \right) \\ &= \left(\frac{2x+2-6}{x+1} \right) \left(\frac{x+1}{x-2} \right) \\ &= \left(\frac{2x-4}{x+1} \right) \left(\frac{x+1}{x-2} \right) \\ &= \left(\frac{\cancel{2(x-2)}}{\cancel{x+1}} \right) \left(\frac{\cancel{x+1}}{\cancel{x-2}} \right) \\ &= \boxed{2, x \neq 2, -1}\end{aligned}$$

(d)

$$\begin{aligned}\left(4 - \frac{3}{x+2}\right)\left(1 + \frac{5}{x-1}\right) &= \left(4 \cdot \frac{x+2}{x+2} - \frac{3}{x+2}\right)\left(1 \cdot \frac{x-1}{x-1} + \frac{5}{x-1}\right) \\ &= \left(\frac{4(x+2)}{x+2} - \frac{3}{x+2}\right)\left(\frac{x-1}{x-1} + \frac{5}{x-1}\right) \\ &= \left(\frac{4(x+2)-3}{x+2}\right)\left(\frac{x-1+5}{x-1}\right) \\ &= \left(\frac{4x+8-3}{x+2}\right)\left(\frac{x+4}{x-1}\right) \\ &= \left(\frac{4x+5}{x+2}\right)\left(\frac{x+4}{x-1}\right) \\ &= \boxed{\frac{(4x+5)(x+4)}{(x+2)(x-1)}, x \neq -2, 1}\end{aligned}$$

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