Math 134 Spring 2018
Krystin Manguba-Glover
Classwork 7   Name: _____

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Solve and check the linear equation.

   (a) $5x - (3x - 8) = 26$

   (b) $3(x - 2) + 8 = 2(x + 5)$

   (c) $10 - \dfrac{x}{2} = \dfrac{x}{3}$

   (d) $\dfrac{11x}{12} = \dfrac{x}{2} + 5$

   Solution

   (a)

   $$5x - (3x - 8) = 26 \Leftrightarrow 5x - 3x + 8 = 26$$
   $$\Leftrightarrow 2x + 8 = 26$$
   $$\Leftrightarrow 2x = 18$$
   $$\Leftrightarrow \boxed{x = 9}$$

   Check:

   $$5(9) - (3(9) - 8) = 26 \Leftrightarrow 45 - (27 - 8) = 26$$
   $$\Leftrightarrow 45 - 19 = 26$$
   $$\Leftrightarrow 26 = 26\checkmark$$

   (b)

   $$3(x - 2) + 8 = 2(x + 5) \Leftrightarrow 3x - 6 + 8 = 2x + 10$$
   $$\Leftrightarrow 3x - 2x = 10 + 6 - 8$$
   $$\Leftrightarrow \boxed{x = 8}$$

   Check:

   $$3(8 - 2) + 8 = 2(8 + 5) \Leftrightarrow 3(6) + 8 = 2(13)$$
   $$\Leftrightarrow 18 + 8 = 26$$
   $$\Leftrightarrow 26 = 26\checkmark$$

   (c)

   $$10 - \frac{x}{2} = \frac{x}{3} \Leftrightarrow 6\left(10 - \frac{x}{2}\right) = 6\left(\frac{x}{3}\right)$$
   $$\Leftrightarrow 60 - 3x = 2x$$
   $$\Leftrightarrow 60 = 5x$$
   $$\Leftrightarrow \boxed{x = 12}$$

<u>Check:</u>

$$10 - \frac{12}{2} = \frac{12}{3} \Leftrightarrow 10 - 6 = 4$$

$$\Leftrightarrow 4 = 4\checkmark$$

(d)

$$\frac{11x}{12} = \frac{x}{2} + 5 \Leftrightarrow 12\left(\frac{11x}{12}\right) = 12\left(\frac{x}{2} + 5\right)$$

$$\Leftrightarrow 11x = 6x + 60$$
$$\Leftrightarrow 5x = 60$$
$$\Leftrightarrow \boxed{x = 12}$$

<u>Check:</u>

$$\frac{11(12)}{12} = \frac{12}{2} + 5 \Leftrightarrow 11 = 6 + 5$$

$$\Leftrightarrow 11 = 11\checkmark$$

□

2. Complete parts (i) and (ii) for the following equations:

   (i) Find the value(s) of the variable that make the denominator zero. These are the restrictions on the variable.

   (ii) Keeping these restrictions in mind, solve the equation.

   (a) $\dfrac{4}{5x + 25} = \dfrac{8}{x + 5} - \dfrac{2}{5}$

   (b) $\dfrac{2x}{x + 2} = 8 - \dfrac{4}{x + 2}$

   (c) $\dfrac{1}{x - 3} - \dfrac{5}{x + 7} = \dfrac{10}{x^2 + 4x - 21}$

   Solution

   (a) (i)

   $$5x + 25 \neq 0 \Leftrightarrow 5x \neq -25$$
   $$\Leftrightarrow x \neq -5$$

   $$x + 5 \neq 0 \Leftrightarrow x \neq -5$$

   Thus $\boxed{x \neq -5}$

   (ii)

   $$\frac{4}{5x + 25} = \frac{8}{x + 5} - \frac{2}{5} \Leftrightarrow \frac{4}{5(x + 5)} = \frac{8}{x + 5} - \frac{2}{5}$$

   $$\Rightarrow 5(x + 5)\left(\frac{4}{5(x + 5)}\right) = 5(x + 5)\left(\frac{8}{x + 5} - \frac{2}{5}\right)$$

   $$\Leftrightarrow \frac{4(5\cancel{(x+5)})}{\cancel{5(x+5)}} = \frac{8(5\cancel{(x+5)})}{\cancel{x+5}} - \frac{2(\cancel{5}(x+5))}{\cancel{5}}$$

   $$\Leftrightarrow 4 = 8 \cdot 5 - 2(x + 5)$$
   $$\Leftrightarrow 4 = 40 - 2x - 10$$
   $$\Leftrightarrow 4 = 30 - 2x$$
   $$\Leftrightarrow 2x = 26$$
   $$\Leftrightarrow \boxed{x = 13}$$

   (b) (i)

   $$x + 2 \neq 0 \Leftrightarrow \boxed{x \neq -2}$$

(ii)

$$\frac{2x}{x+2} = 8 - \frac{4}{x+2} \Rightarrow (x+2)\left(\frac{2x}{x+2}\right) = (x+2)\left(8 - \frac{4}{x+2}\right)$$

$$\Leftrightarrow \frac{2x\cancel{(x+2)}}{\cancel{x+2}} = 8(x+2) - \frac{4\cancel{(x+2)}}{\cancel{x+2}}$$

$$\Leftrightarrow 2x = 8(x+2) - 4$$

$$\Leftrightarrow 2x = 8x + 16 - 4$$

$$\Leftrightarrow 2x = 8x + 12$$

$$\Leftrightarrow 6x = -12$$

$$\Leftrightarrow x = -2$$

Thus there is $\boxed{\text{no solution}}$

(c) (i)

$$x - 3 \neq 0 \Leftrightarrow x \neq 3$$

$$x + 7 \neq 0 \Leftrightarrow x \neq -7$$

$$x^2 + 4x - 21 \neq 0 \Leftrightarrow (x+7)(x-3) \neq 0$$
$$\Leftrightarrow x \neq -7, 3$$

Thus $\boxed{x \neq -7, 3}$

(ii)

$$\frac{1}{x-3} - \frac{5}{x+7} = \frac{10}{x^2+4x-21} \Rightarrow \frac{1}{x-3} - \frac{5}{x+7} = \frac{10}{(x+7)(x-3)}$$

$$\Leftrightarrow (x-3)(x+7)\left(\frac{1}{x-3} - \frac{5}{x+7}\right) = (x-3)(x+7)\left(\frac{10}{(x+7)(x-3)}\right)$$

$$\Leftrightarrow \frac{\cancel{(x-3)}(x+7)}{\cancel{x-3}} - \frac{5(x-3)\cancel{(x+7)}}{\cancel{x+7}} = \frac{10\cancel{(x-3)}\cancel{(x+7)}}{\cancel{(x+7)}\cancel{(x-3)}}$$

$$\Leftrightarrow (x+7) - 5(x-3) = 10$$

$$\Leftrightarrow x + 7 - 5x + 15 = 10$$

$$\Leftrightarrow -4x = 10 - 15 - 7$$

$$\Leftrightarrow -4x = -12$$

$$\Leftrightarrow x = 3$$

Thus there is $\boxed{\text{no solution}}$

$\square$

3. Solve the following equations. Determine whether each equation is an identity, conditional equation, or an inconsistent equation.

(a) $5x + 24 = 8(x + 3) - 3x$

(b) $\dfrac{x + 6}{2} - 7 = \dfrac{2x - 8}{3}$

(c) $\dfrac{6x}{x + 2} - \dfrac{12}{x - 2} = \dfrac{6x^2 + 24}{x^2 - 4}$

Solution

(a)

$$5x + 24 = 8(x + 3) - 3x \Leftrightarrow 5x + 24 = 8x + 24 - 3x$$
$$\Leftrightarrow 5x + 24 = 5x + 24$$

Thus it is a $\boxed{\text{identity equation}}$ and is true for all real x.

(b)

$$\frac{x + 6}{2} - 7 = \frac{2x - 8}{3} \Leftrightarrow 6\left(\frac{x + 6}{2} - 7\right) = 6\left(\frac{2x - 8}{3}\right)$$
$$\Leftrightarrow 3(x + 6) - 42 = 2(2x - 8)$$
$$\Leftrightarrow 3x + 18 - 42 = 4x - 16$$
$$\Leftrightarrow x = 18 - 42 + 16$$
$$\Leftrightarrow \boxed{x = -8}$$

Thus it is a $\boxed{\text{conditional equation}}$

(c) (Note: $x \neq \pm 2$)

$$\frac{6x}{x + 2} - \frac{12}{x - 2} = \frac{6x^2 + 24}{x^2 - 4} \Leftrightarrow \frac{6x}{x + 2} - \frac{12}{x - 2} = \frac{6x^2 + 24}{(x - 2)(x + 2)}$$
$$\Rightarrow (x + 2)(x - 2)\left(\frac{6x}{x + 2} - \frac{12}{x - 2}\right) = (x + 2)(x - 2)\left(\frac{6x^2 + 24}{(x - 2)(x + 2)}\right)$$
$$\Leftrightarrow \frac{6x\cancel{(x + 2)}(x - 2)}{\cancel{x + 2}} - \frac{12(x + 2)\cancel{(x - 2)}}{\cancel{x - 2}} = \frac{\cancel{(x + 2)}\cancel{(x - 2)}(6x^2 + 24)}{\cancel{(x - 2)}\cancel{(x + 2)}}$$
$$\Leftrightarrow 6x(x - 2) - 12(x + 2) = 6x^2 + 24$$
$$\Leftrightarrow 6x^2 - 12x - 12x - 24 = 6x^2 + 24$$
$$\Leftrightarrow -24x = 24 + 24$$
$$\Leftrightarrow -24x = 48$$
$$\Leftrightarrow x = -2$$

Thus there is no solution and it is an $\boxed{\text{inconsistent equation}}$

☐

4. Find the value of x satisfying the given conditions:

(a) $y_1 = 5(3x - 5) - 15$, $y_2 = 6(x - 5) + 8$, $y_1 = y_2$

(b) $y_1 = \dfrac{x - 3}{3}$, $y_2 = \dfrac{x - 9}{4}$, $y_1 - y_2 = 1$

Solution

(a)

$$5(3x - 5) - 15 = 6(x - 5) + 8 \Leftrightarrow 15x - 25 - 15 = 6x - 30 + 8$$
$$\Leftrightarrow 15x - 40 = 6x - 22$$
$$\Leftrightarrow 9x = 18$$
$$\Leftrightarrow \boxed{x = 2}$$

(b)

$$\frac{x - 3}{3} - \frac{x - 9}{4} = 1 \Leftrightarrow 12\left(\frac{x - 3}{3} - \frac{x - 9}{4}\right) = 12$$
$$\Leftrightarrow 4(x - 3) - 3(x - 9) = 12$$
$$\Leftrightarrow 4x - 12 - 3x + 27 = 12$$
$$\Leftrightarrow x + 15 = 12$$
$$\Leftrightarrow \boxed{x = -3}$$

□

5. Solve the following word problems:

   (a) When twice a number is decreased by seven, the result is 33. What is the number?

   (b) When three times a number is decreased by six, the result is 30. What is the number?

   (c) When a number is decreased by 40% of itself, the result is 30. What is the number?

   (d) One number exceeds another number by 1. The sum of the numbers is 57. What are the numbers?

Solution

   (a)

   $$2x - 7 = 33 \Leftrightarrow 2x = 40$$
   $$\Leftrightarrow \boxed{x = 20}$$

   (b)

   $$3x - 6 = 30 \Leftrightarrow 3x = 36$$
   $$\Leftrightarrow \boxed{x = 12}$$

   (c)

   $$x - 0.4x = 30 \Leftrightarrow 0.6x = 30$$
   $$\Leftrightarrow \frac{6}{10}x = 30$$
   $$\Leftrightarrow \frac{3}{5}x = 30$$
   $$\Leftrightarrow x = 30 \cdot \frac{5}{3}$$
   $$\Leftrightarrow x = 10 \cdot 5$$
   $$\Leftrightarrow \boxed{x = 50}$$

   (d)

   $$x + (x + 1) = 57 \Leftrightarrow 2x + 1 = 57$$
   $$\Leftrightarrow 2x = 56$$
   $$\Leftrightarrow x = 28$$

   Thus the two numbers are $\boxed{28 \text{ and } 29}$

   □

6. Find all values of x satisfying the conditions.

   (a) $y_1 = \dfrac{x-4}{4}$, $y_2 = \dfrac{x-12}{5}$, and $y_1 - y_2 = 1$

   (b) $y_1 = 10x - 5$, $y_2 = 6x + 12$, and y_1 exceeds y_2 by 3.

   Solution

   (a)

   $$\frac{x-4}{4} - \frac{x-12}{5} = 1 \Leftrightarrow 20\left(\frac{x-4}{4} - \frac{x-12}{5}\right) = 1$$
   $$\Leftrightarrow 5(x-4) - 4(x-12) = 1$$
   $$\Leftrightarrow 5x - 20 - 4x + 48 = 1$$
   $$\Leftrightarrow x + 28 = 1$$
   $$\Leftrightarrow \boxed{x = -27}$$

   (b)

   $$10x - 5 + 3 = 6x + 12 \Leftrightarrow 10x - 2 = 6x + 12$$
   $$\Leftrightarrow 4x = 16$$
   $$\Leftrightarrow \boxed{x = 4}$$

   $\square$

7. According to statistics, a person will devote 31 years to sleeping and watching TV. The number of years sleeping will exceed the number of years watching TV by 19. Over the lifetime, how many years will the person spend watching TV? How many years will they spend sleeping?

   Solution   Let x be the years devoted to watching TV. We have

   $$x + (x + 19) = 31 \Leftrightarrow 2x + 19 = 31$$
   $$\Leftrightarrow 2x = 12$$
   $$\Leftrightarrow x = 6$$

   Thus the person will spend $\boxed{6 \text{ years watching TV and 25 years sleeping}}$

   $\square$

8. The length of a new rectangular playing field is 9 yards longer than double the width. If the perimeter of the rectangular playing field is 360 yards, what are its dimensions?

Solution   Let x be the width of the field. We have

$$2(2x + 9) + 2x = 360 \Leftrightarrow 4x + 18 + 2x = 360$$
$$\Leftrightarrow 6x + 18 = 360$$
$$\Leftrightarrow 6x = 342$$
$$\Leftrightarrow x = 57$$

Thus the dimensions are $\boxed{57 \text{ yards by } 123 \text{ yards}}$

☐

9. City Cabs charges a \$2.75 pickup fee and \$1.50 per mile traveled. Diego's fare is \$17.75. How far did he travel in the cab?

Solution

$$2.75 + 1.50x = 17.75 \Leftrightarrow 1.50x = 15$$
$$\Leftrightarrow x = 10$$

Thus Diego travelled $\boxed{10 \text{ miles}}$

☐

10. Solve for the specified variable:

(a) $v = gh$ for g

(b) $B = \dfrac{1}{4}h(q + z)$ for q

(c) $m = \dfrac{t}{f - w}$ for f

Solution

(a)

$$v = gh \Leftrightarrow \boxed{g = \dfrac{v}{h}}$$

(b)

$$B = \dfrac{1}{4}h(q + z) \Leftrightarrow \dfrac{4B}{h} = q + z$$

$$\Leftrightarrow \boxed{q = \dfrac{4B}{h} - z}$$

(c)

$$m = \dfrac{t}{f - w} \Leftrightarrow m(f - w) = t$$

$$\Leftrightarrow f - w = \dfrac{t}{m}$$

$$\Leftrightarrow \boxed{f = \dfrac{t}{m} + w}$$

$\square$