

Name: _____

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Perform the operation of complex numbers. Write the result in standard form:

- (a) $(7 + 3i) - (-4 + 5i)$
- (b) $(6 - 5i) + (14 - 3i) - (7 + i)$
- (c) $(19 + i) + 7i - (3 - 4i) + 2$
- (d) $i + 3 + (i - 3) + (3i - 1)$
- (e) $2 - 3i + (4i - 5i) + 6i - (7i - 2)$

Solution

(a)

$$\begin{aligned}(7 + 3i) - (-4 + 5i) &= 7 + 3i + 4 - 5i \\ &= \boxed{11 - 2i}\end{aligned}$$

(b)

$$\begin{aligned}(6 - 5i) + (14 - 3i) - (7 + i) &= 6 - 5i + 14 - 3i - 7 - i \\ &= \boxed{13 - 9i}\end{aligned}$$

(c)

$$\begin{aligned}(19 + i) + 7i - (3 - 4i) + 2 &= 19 + i + 7i - 3 + 4i + 2 \\ &= \boxed{18 + 12i}\end{aligned}$$

(d)

$$\begin{aligned}i + 3 + (i - 3) + (3i - 1) &= i + 3 + i - 3 + 3i - 1 \\ &= \boxed{-1 + 5i}\end{aligned}$$

(e)

$$\begin{aligned}2 - 3i + (4i - 5i) + 6i - (7i - 2) &= 2 - 3i + 4i - 5i + 6i - 7i + 2 \\ &= \boxed{-5i + 4}\end{aligned}$$

□

2. Perform the operation of complex numbers. Write the result in standard form:

(a) $-9i^2(3i - 5i^2)$

(b) $7i(-4 - 3i)$

(c) $i(4 + i)(1 + i)$

(d) $(2 + 3i)(7 - 2i)$

(e) $(3 - 8i)(2 + 7i)$

Solution

(a)

$$\begin{aligned} -9i^2(3i - 5i^2) &= 9(3i + 5) \\ &= 27i + 45 \\ &= \boxed{45 + 27i} \end{aligned}$$

(b)

$$\begin{aligned} 7i(-4 - 3i) &= -28i - 21i^2 \\ &= -28i + 21 \\ &= \boxed{21 - 28i} \end{aligned}$$

(c)

$$\begin{aligned} i(4 + i)(1 + i) &= (4i + i^2)(1 + i) \\ &= (4i - 1)(1 + i) \\ &= 4i + 4i^2 - 1 - i \\ &= 4i - 4 - 1 - i \\ &= \boxed{-5 + 3i} \end{aligned}$$

(d)

$$\begin{aligned} (2 + 3i)(7 - 2i) &= 14 - 4i + 21i - 6i^2 \\ &= 14 - 4i + 21i + 6 \\ &= \boxed{20 + 17i} \end{aligned}$$

(e)

$$\begin{aligned} (3 - 8i)(2 + 7i) &= 6 + 21i - 16i - 56i^2 \\ &= 6 + 21i - 16i + 56 \\ &= \boxed{62 + 5i} \end{aligned}$$

□

3. Perform the operation of complex numbers. Write the result in standard form:

(a) $\frac{4-3i}{5+5i}$

(b) $\frac{17-8i}{-5i}$

(c) $\frac{3+4i}{3-4i}$

(d) $\frac{5i}{3-4i}$

(e) $\frac{9-2i}{3+4i}$

Solution

(a)

$$\begin{aligned}\frac{4-3i}{5+5i} &= \frac{4-3i}{5+5i} \cdot \frac{5-5i}{5-5i} \\ &= \frac{(4-3i)(5-5i)}{(5+5i)(5-5i)} \\ &= \frac{20-20i-15i+15i^2}{25-25i+25i-25i^2} \\ &= \frac{20-35i-15}{25+25} \\ &= \frac{5-35i}{50} \\ &= \frac{5}{50} - \frac{35}{50}i \\ &= \boxed{\frac{1}{10} - \frac{7}{10}i}\end{aligned}$$

(b)

$$\begin{aligned}\frac{17-8i}{-5i} &= \frac{17-8i}{-5i} \cdot \frac{5i}{5i} \\ &= \frac{(17-8i)(5i)}{(-5i)(5i)} \\ &= \frac{85i-40i^2}{-25i^2} \\ &= \frac{85i+40}{25} \\ &= \frac{40}{25} + \frac{85}{25}i \\ &= \boxed{\frac{8}{5} + \frac{17}{5}i}\end{aligned}$$

(c)

$$\begin{aligned}\frac{3+4i}{3-4i} &= \frac{3+4i}{3-4i} \cdot \frac{3+4i}{3+4i} \\ &= \frac{(3+4i)(3+4i)}{(3-4i)(3+4i)} \\ &= \frac{9+12i+12i+16i^2}{9+12i-12i-16i^2} \\ &= \frac{9+24i-16}{9+16} \\ &= \frac{-7+24i}{25} \\ &= \boxed{-\frac{7}{25} + \frac{24}{25}i}\end{aligned}$$

(d)

$$\begin{aligned}\frac{5i}{3-4i} &= \frac{5i}{3-4i} \cdot \frac{3+4i}{3+4i} \\ &= \frac{5i(3+4i)}{(3-4i)(3+4i)} \\ &= \frac{15i+20i^2}{9+12i-12i-16i^2} \\ &= \frac{15i-20}{9+16} \\ &= \frac{-20+15i}{25} \\ &= -\frac{20}{25} + \frac{15}{25}i \\ &= \boxed{-\frac{4}{5} + \frac{3}{5}i}\end{aligned}$$

(e)

$$\begin{aligned}\frac{9-2i}{3+4i} &= \frac{9-2i}{3+4i} \cdot \frac{3-4i}{3-4i} \\ &= \frac{(9-2i)(3-4i)}{(3+4i)(3-4i)} \\ &= \frac{27-36i-6i+8i^2}{9-12i+12i-16i^2} \\ &= \frac{27-42i-8}{9+16} \\ &= \frac{19-42i}{25} \\ &= \boxed{\frac{19}{25} - \frac{42}{25}i}\end{aligned}$$

□

4. Perform the operation of complex numbers. Write the result in standard form:

(a) $\sqrt{-50} - \sqrt{-8}$

(b) $\sqrt{-3}(\sqrt{-75} - \sqrt{3})$

(c) $\sqrt{-8} - \sqrt{-18} + \sqrt{-32}$

(d) $\sqrt{(3 + \sqrt{-16})(3 - \sqrt{-16})}$

(e) $\sqrt{-32} + \sqrt[3]{-27} - \sqrt{-16}$

Solution

(a)

$$\begin{aligned}\sqrt{-50} - \sqrt{-8} &= i\sqrt{50} - i\sqrt{8} \\ &= i\sqrt{25 \cdot 2} - i\sqrt{4 \cdot 2} \\ &= 5i\sqrt{2} - 2i\sqrt{2} \\ &= \boxed{3i\sqrt{2}}\end{aligned}$$

(b)

$$\begin{aligned}\sqrt{-3}(\sqrt{-75} - \sqrt{3}) &= i\sqrt{3}(i\sqrt{75} - \sqrt{3}) \\ &= i\sqrt{3}(i\sqrt{25 \cdot 3} - \sqrt{3}) \\ &= i\sqrt{3}(5i\sqrt{3} - \sqrt{3}) \\ &= 5i^2 \cdot 3 - 3i \\ &= \boxed{-15 - 3i}\end{aligned}$$

(c)

$$\begin{aligned}\sqrt{-8} - \sqrt{-18} + \sqrt{-32} &= i\sqrt{8} - i\sqrt{18} + i\sqrt{32} \\ &= i\sqrt{4 \cdot 2} - i\sqrt{9 \cdot 2} + i\sqrt{16 \cdot 2} \\ &= 2i\sqrt{2} - 3i\sqrt{2} + 4i\sqrt{2} \\ &= \boxed{3i\sqrt{2}}\end{aligned}$$

(d)

$$\begin{aligned}\sqrt{(3 + \sqrt{-16})(3 - \sqrt{-16})} &= \sqrt{(3 + i\sqrt{16})(3 - i\sqrt{16})} \\ &= \sqrt{(3 + 4i)(3 - 4i)} \\ &= \sqrt{(9 - 12i + 12i - 16i^2)} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= \boxed{5}\end{aligned}$$

(e)

$$\begin{aligned}\sqrt{-32} + \sqrt[3]{-27} - \sqrt{-16} &= i\sqrt{32} - 3 - i\sqrt{16} \\ &= i\sqrt{16 \cdot 2} - 3 - 4i \\ &= 4i\sqrt{2} - 3 - 4i \\ &= \boxed{-3 + (4\sqrt{2} - 4)i}\end{aligned}$$

□

5. Perform the operation of complex numbers. Write the result in standard form:

- (a) i^{13}
- (b) $-i^{17}$
- (c) $(1+i)^3$
- (d) $(2i)^5 + i^9$
- (e) $4i^3 - 3i^2 + 2i - 1$

Solution

(a)

$$\begin{aligned}i^{13} &= i^{12}i \\ &= (i^4)^3i \\ &= (1)^3i \\ &= \boxed{i}\end{aligned}$$

(b)

$$\begin{aligned}-i^{17} &= -i^{16}i \\ &= -(i^4)^4i \\ &= -(1)^4i \\ &= \boxed{-i}\end{aligned}$$

(c)

$$\begin{aligned}(1+i)^3 &= (1+i)(1+i)(1+i) \\ &= (1+i+i+i^2)(1+i) \\ &= (1+2i-1)(1+i) \\ &= 2i(1+i) \\ &= 2i+2i^2 \\ &= \boxed{-2+2i}\end{aligned}$$

(d)

$$\begin{aligned}(2i)^5 + i^9 &= 2^5i^5 + i^8i \\ &= 32i^4i + (i^4)^2i \\ &= 32(1)i + (1)^2i \\ &= 32i + i \\ &= \boxed{33i}\end{aligned}$$

(e)

$$\begin{aligned}4i^3 - 3i^2 + 2i - 1 &= 4(-i) + 3 + 2i - 1 \\ &= -4i + 3 + 2i \\ &= \boxed{3 - 2i}\end{aligned}$$

□