

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present a problem correctly to receive classwork points.

1. Solve the following equations by factoring.

(a) $x^2 = 2x + 15$

(b) $6x^2 + 5x - 6 = 0$

(c) $x^2 + 11x + 24 = 0$

(d) $x^2 + 4x + 3 = 0$

(e) $2x^2 - 5x = 3$

Solution

(a)

$$\begin{aligned}x^2 = 2x + 15 &\Leftrightarrow x^2 - 2x - 15 = 0 \\&\Leftrightarrow (x - 5)(x + 3) = 0 \\&\Leftrightarrow x - 5 = 0 \text{ or } x + 3 = 0 \\&\Leftrightarrow \boxed{x = 5 \text{ or } -3}\end{aligned}$$

(b)

$$\begin{aligned}6x^2 + 5x - 6 = 0 &\Leftrightarrow (3x - 2)(2x + 3) = 0 \\&\Leftrightarrow 3x - 2 = 0 \text{ or } 2x + 3 = 0 \\&\Leftrightarrow 3x = 2 \text{ or } 2x = -3 \\&\Leftrightarrow \boxed{x = \frac{2}{3} \text{ or } x = -\frac{3}{2}}\end{aligned}$$

(c)

$$\begin{aligned}x^2 + 11x + 24 = 0 &\Leftrightarrow (x + 8)(x + 3) = 0 \\&\Leftrightarrow x + 8 = 0 \text{ or } x + 3 = 0 \\&\Leftrightarrow \boxed{x = -8 \text{ or } x = -3}\end{aligned}$$

(d)

$$\begin{aligned}x^2 + 4x + 3 = 0 &\Leftrightarrow (x + 3)(x + 1) = 0 \\&\Leftrightarrow x + 3 = 0 \text{ or } x + 1 = 0 \\&\Leftrightarrow \boxed{x = -3 \text{ or } x = -1}\end{aligned}$$

(e)

$$\begin{aligned}2x^2 - 5x = 3 &\Leftrightarrow 2x^2 - 5x - 3 = 0 \\&\Leftrightarrow (2x + 1)(x - 3) = 0 \\&\Leftrightarrow 2x + 1 = 0 \text{ or } x - 3 = 0 \\&\Leftrightarrow 2x = -1 \text{ or } x = 3 \\&\Leftrightarrow \boxed{x = -\frac{1}{2} \text{ or } x = 3}\end{aligned}$$

□

2. Solve the following equations by using the square root property.

(a) $4x^2 + 3 = 19$

(b) $2x^2 + 7 = 207$

(c) $x^2 - 36 = 0$

(d) $4x^2 + 2 = 258$

(e) $9x^2 - 49 = 0$

Solution

(a)

$$4x^2 + 3 = 19 \Leftrightarrow 4x^2 = 16$$

$$\Leftrightarrow x^2 = 4$$

$$\Leftrightarrow \boxed{x = \pm 2}$$

(b)

$$2x^2 + 7 = 207 \Leftrightarrow 2x^2 = 200$$

$$\Leftrightarrow x^2 = 100$$

$$\Leftrightarrow \boxed{x = \pm 10}$$

(c)

$$x^2 - 36 = 0 \Leftrightarrow x^2 = 36$$

$$\Leftrightarrow \boxed{x = \pm 6}$$

(d)

$$4x^2 + 2 = 258 \Leftrightarrow 4x^2 = 256$$

$$\Leftrightarrow x^2 = 64$$

$$\Leftrightarrow \boxed{x = \pm 8}$$

(e)

$$9x^2 - 49 = 0 \Leftrightarrow 9x^2 = 49$$

$$\Leftrightarrow x^2 = \frac{49}{9}$$

$$\Leftrightarrow \boxed{x = \pm \frac{7}{3}}$$

□

3. Solve the following quadratic equations by completing the square.

- (a) $x^2 + 4x = -3$
- (b) $x^2 + 18x = -56$
- (c) $x^2 + 8x = 33$
- (d) $x^2 - 2x = 5$
- (e) $x^2 - 6x - 4 = 0$

Solution

(a)

$$\begin{aligned}x^2 + 4x = -3 &\Leftrightarrow x^2 + 4x + \left(\frac{4}{2}\right)^2 = -3 + \left(\frac{4}{2}\right)^2 \\&\Leftrightarrow x^2 + 4x + 4 = -3 + 4 \\&\Leftrightarrow (x + 2)^2 = 1 \\&\Leftrightarrow x + 2 = 1 \text{ or } x + 2 = -1 \\&\Leftrightarrow \boxed{x = -1 \text{ or } x = -3}\end{aligned}$$

(b)

$$\begin{aligned}x^2 + 18x = -56 &\Leftrightarrow x^2 + 18x + \left(\frac{18}{2}\right)^2 = -56 + \left(\frac{18}{2}\right)^2 \\&\Leftrightarrow x^2 + 18x + 81 = -56 + 81 \\&\Leftrightarrow (x + 9)^2 = 25 \\&\Leftrightarrow x + 9 = 5 \text{ or } x + 9 = -5 \\&\Leftrightarrow \boxed{x = -4 \text{ or } x = -14}\end{aligned}$$

(c)

$$\begin{aligned}x^2 + 8x = 33 &\Leftrightarrow x^2 + 8x + \left(\frac{8}{2}\right)^2 = 33 + \left(\frac{8}{2}\right)^2 \\&\Leftrightarrow x^2 + 8x + 16 = 33 + 16 \\&\Leftrightarrow (x + 4)^2 = 49 \\&\Leftrightarrow x + 4 = 7 \text{ or } x + 4 = -7 \\&\Leftrightarrow \boxed{x = 3 \text{ or } x = -11}\end{aligned}$$

(d)

$$\begin{aligned}x^2 - 2x = 5 &\Leftrightarrow x^2 - 2x + \left(\frac{-2}{2}\right)^2 = 5 + \left(\frac{-2}{2}\right)^2 \\&\Leftrightarrow x^2 - 2x + 1 = 5 + 1 \\&\Leftrightarrow (x - 1)^2 = 6 \\&\Leftrightarrow x - 1 = \sqrt{6} \text{ or } x - 1 = -\sqrt{6} \\&\Leftrightarrow \boxed{x = 1 + \sqrt{6} \text{ or } x = 1 - \sqrt{6}}\end{aligned}$$

(e)

$$\begin{aligned}x^2 - 6x - 4 = 0 &\Leftrightarrow x^2 - 6x = 4 \\&\Leftrightarrow x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 4 + \left(\frac{-6}{2}\right)^2 \\&\Leftrightarrow x^2 - 6x + 9 = 4 + 9 \\&\Leftrightarrow (x - 3)^2 = 13 \\&\Leftrightarrow x - 3 = \sqrt{13} \text{ and } x - 3 = -\sqrt{13} \\&\Leftrightarrow \boxed{x = 3 + \sqrt{13} \text{ or } x = 3 - \sqrt{13}}\end{aligned}$$

□

4. Solve the following equations using the quadratic formula.

(a) $x^2 + 11x + 30 = 0$

(b) $x^2 + 11x + 4 = 0$

(c) $2x^2 - 3x - 1 = 0$

(d) $x^2 - 8x + 52 = 0$

(e) $x^2 - 10x + 34 = 0$

Solution

(a)

$$x^2 + 11x + 30 = 0 \Leftrightarrow x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(30)}}{2(1)}$$

$$\Leftrightarrow x = \frac{-11 \pm \sqrt{121 - 120}}{2}$$

$$\Leftrightarrow x = \frac{-11 \pm \sqrt{1}}{2}$$

$$\Leftrightarrow x = \frac{-11 \pm 1}{2}$$

$$\Leftrightarrow x = \frac{-11 + 1}{2} \text{ or } x = \frac{-11 - 1}{2}$$

$$\Leftrightarrow x = \frac{-10}{2} \text{ or } x = \frac{-12}{2}$$

$$\Leftrightarrow \boxed{x = -5 \text{ or } x = -6}$$

(b)

$$x^2 + 11x + 4 = 0 \Leftrightarrow x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(4)}}{2(1)}$$

$$\Leftrightarrow x = \frac{-11 \pm \sqrt{121 - 16}}{2}$$

$$\Leftrightarrow \boxed{x = \frac{-11 \pm \sqrt{105}}{2}}$$

(c)

$$2x^2 - 3x - 1 = 0 \Leftrightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$\Leftrightarrow x = \frac{3 \pm \sqrt{9 + 8}}{4}$$

$$\Leftrightarrow \boxed{x = \frac{3 \pm \sqrt{17}}{4}}$$

(d)

$$x^2 - 8x + 52 = 0 \Leftrightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(52)}}{2(1)}$$

$$\Leftrightarrow x = \frac{8 \pm \sqrt{64 - 208}}{2}$$

$$\Leftrightarrow x = \frac{8 \pm \sqrt{-144}}{2}$$

$$\Leftrightarrow x = \frac{8 \pm i\sqrt{144}}{2}$$

$$\Leftrightarrow x = \frac{8 \pm 12i}{2}$$

$$\Leftrightarrow \boxed{x = 4 \pm 6i}$$

(e)

$$x^2 - 10x + 34 = 0 \Leftrightarrow x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(34)}}{2(1)}$$

$$\Leftrightarrow x = \frac{10 \pm \sqrt{100 - 136}}{2}$$

$$\Leftrightarrow x = \frac{10 \pm \sqrt{-36}}{2}$$

$$\Leftrightarrow x = \frac{10 \pm i\sqrt{36}}{2}$$

$$\Leftrightarrow x = \frac{10 \pm 6i}{2}$$

$$\Leftrightarrow \boxed{x = 5 \pm 3i}$$

□

5. Compute the discriminant and determine the number and type of solutions of the given equation.

(a) $8x^2 - 10x + 4$

(b) $5x^2 + 3x - 1$

(c) $x^2 + 7x + 5$

(d) $-x^2 + 2x - 1$

(e) $x^2 - 2x - 7$

Solution

(a)

$$\begin{aligned}\text{Discriminant} &= (-10)^2 - 4(8)(4) \\ &= 100 - 128 \\ &= \boxed{-28} < 0\end{aligned}$$

Thus there are two complex solutions

(b)

$$\begin{aligned}\text{Discriminant} &= (3)^2 - 4(5)(-1) \\ &= 9 + 20 \\ &= \boxed{29} > 0\end{aligned}$$

Thus there are two real solutions

(c)

$$\begin{aligned}\text{Discriminant} &= (7)^2 - 4(1)(5) \\ &= 49 - 20 \\ &= \boxed{29} > 0\end{aligned}$$

Thus there are two real solutions

(d)

$$\begin{aligned}\text{Discriminant} &= (2)^2 - 4(-1)(-1) \\ &= 4 - 4 \\ &= 0\end{aligned}$$

Thus there is one real solution

(e)

$$\begin{aligned}\text{Discriminant} &= (-2)^2 - 4(1)(-7) \\ &= 4 + 28 \\ &= \boxed{32} > 0\end{aligned}$$

Thus there are two real solutions

□