Final Exam Review Questions

1. Solve the linear equation

$$6x + 7 = 3x + 40$$

Solution

$$\begin{aligned} 6x + 7 = 3x + 40 &\Leftrightarrow 3x = 33 \\ &\Leftrightarrow \boxed{x = 11} \end{aligned}$$

□

2. Solve the formula for p:

$$S = W + pg$$

Solution

$$\begin{aligned} S = W + pg &\Leftrightarrow pg = S - W \\ &\Leftrightarrow \boxed{p = \frac{S - W}{g}} \end{aligned}$$

□

3. Solve the equation. Then determine whether the equation is an identity, a conditional equation, or an inconsistent equation.

$$5(x + 3) = 3 + 5x$$

Solution

$$\begin{aligned} 5(x + 3) = 3 + 5x &\Leftrightarrow 5x + 15 = 3 + 5x \\ &\Leftrightarrow 15 = 3 \end{aligned}$$

So the equation is $\boxed{\text{inconsistent}}$

□

4. Graph the equation

$$y = |x| - 3$$

(a) Determine the x-intercepts, if any.

(b) Determine the y-intercepts, if any.

Solution Use a graphing utility to check your graph. It is the graph of $|x|$ shifted down 3 units.

(a) $\boxed{\pm 3}$

(b) $\boxed{-3}$

$\square$

5. Consider the equation

$$\frac{1}{x-4} + 3 = \frac{16}{x-4}$$

(a) Write the value(s) of x that make the denominator zero (i.e.: the restrictions).

(b) Keeping the restrictions in mind, solve the equation.

Solution

(a) $\boxed{x = 4}$

(b)

$$\frac{1}{x-4} + 3 = \frac{16}{x-4} \Rightarrow (x-4)\left(\frac{1}{x-4} + 3\right) = (x-4) \cdot \frac{16}{x-4}$$

$$\Leftrightarrow 1 + 3(x-4) = 16$$

$$\Leftrightarrow 3x - 11 = 16$$

$$\Leftrightarrow 3x = 27$$

$$\Leftrightarrow \boxed{x = 9}$$

$\square$

6. Solve the quadratic equation by completing the square.

$$x^2 - 2x = 2$$

Solution

$$\begin{aligned} x^2 - 2x = 2 &\Leftrightarrow x^2 - 2x + \left(2/2\right)^2 = 2 + \left(2/2\right)^2 \\ &\Leftrightarrow x^2 - 2x + 1 = 2 + 1 \\ &\Leftrightarrow (x-1)^2 = 3 \\ &\Leftrightarrow x - 1 = \pm\sqrt{3} \\ &\Leftrightarrow \boxed{x = 1 \pm \sqrt{3}} \end{aligned}$$

□

7. Solve the following equation using the quadratic formula.

$$x^2 + 11x + 30 = 0$$

Solution

$$\begin{aligned} x &= \frac{-11 \pm \sqrt{121 - 4(1)(30)}}{2(1)} \\ &= \frac{-11 \pm \sqrt{1}}{2} \\ &= \boxed{-5, -6} \end{aligned}$$

□

8. Compute the discriminant. Then determine the number and type of solutions of the given equation. (You do not need to find the solutions!)

$$2x^2 - 6x + 5 = 0$$

Solution

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (-6)^2 - 4(2)(5) \\ &= \boxed{-2} \end{aligned}$$

Thus there are $\boxed{\text{two complex solutions}}$

□

9. Find all the roots. (solve by making the appropriate substitution.)

$$x^4 - 11x^2 + 18 = 0$$

Solution Let $u = x^2$

$$\begin{aligned} x^4 - 11x^2 + 18 = 0 &\Leftrightarrow u^2 - 11u + 18 = 0 \\ &\Leftrightarrow (u-2)(u-9) = 0 \\ &\Leftrightarrow u = 2, 9 \\ &\Leftrightarrow \boxed{x = \pm\sqrt{2}, \pm 3} \end{aligned}$$

□

10. Solve the equation.

$$5x^{3/2} = 135$$

Solution

$$\begin{aligned} 5x^{3/2} = 135 &\Leftrightarrow x^{3/2} = 27 \\ &\Leftrightarrow x = 27^{2/3} \\ &\Leftrightarrow x = (\sqrt[3]{27})^2 \\ &\Leftrightarrow x = 3^2 \\ &\Leftrightarrow \boxed{x = 9} \end{aligned}$$

□

11. Find the real solutions of the equation. Check your solutions!

$$\sqrt{36 - 5x} = x$$

Solution

$$\begin{aligned} \sqrt{36 - 5x} = x &\Rightarrow 36 - 5x = x^2 \\ &\Leftrightarrow x^2 + 5x - 36 = 0 \\ &\Leftrightarrow (x+9)(x-4) = 0 \\ &\Leftrightarrow x = -9, 4 \end{aligned}$$

Check:

$$\sqrt{36 - 5(4)} = 4 \checkmark \sqrt{36 - 5(-9)} = 9 \times$$

So the solution is just $\boxed{x = 4}$

□

12. Find the solutions(s) of the equation

$$|2x - 3| = 11$$

Solution

$$2x - 3 = 11 \text{ or } 2x - 3 = -11$$

$$2x = 14 \text{ or } 2x = -8$$

$$\boxed{x = 7 \text{ or } x = -4}$$

□

13. Solve the absolute value inequality.

$$8 \le |4x - 4|$$

Solution

$$8 \le 4x - 4 \text{ or } -8 \ge 4x - 4$$

$$12 \le 4x \text{ or } -4 \ge 4x$$

$$\boxed{3 \le x \text{ or } -1 \ge x}$$

□

14. According to statistics, a person will devote 37 years to sleeping and watching TV. The number of years sleeping will exceed the number of years watching TV by 19. Over the lifetime, how many years will the person spend on each of these activities?

Solution Let S be the amount of time spent sleeping and T be the amount of time spent watching TV. We know that

$$S + TV = 37 \text{ and } S = T + 19$$

Plugging $S = T + 19$ into the first equations we have

$$(T + 19) + T = 37 \Leftrightarrow 2T + 19 = 37 \Leftrightarrow 2T = 18 \Leftrightarrow \boxed{T = 9 \Rightarrow S = 28}$$

□

15. On two examinations, you have grades of 84 and 89. There is an optional final examination, which counts as one grade. You decide to take the final in order to get a course grade of A, meaning a final average of at least 90.

    (a) What must you get on the final to earn an A in the course?

    (b) By taking the final, if you do poorly, you might risk the *B* that you have in the course based on the first two exam grades. If your final average is less than 80, you will lose your B in the course. Describe the grades on the final that will cause this to happen.

Solution Let x be the score you receive on the final exam. Then the average of the three grades is given by

$$\frac{84+89+x}{3}$$

(a)

$$\frac{84+89+x}{3} \geq 90 \Leftrightarrow 173+x \geq 270 \Leftrightarrow x \geq 97$$

You must get $\boxed{\text{at least a 97}}$

(b)

$$\frac{84+89+x}{3} < 80 \Leftrightarrow 173+x < 240 \Leftrightarrow x < 67$$

If you get $\boxed{\text{less than a 67}}$

□

16. Solve the quadratic equation by completing the square.

$$ax^2 + bx + c = 0$$

Solution Assume $a > 0$ (otherwise multiply the equation by -1)

$$\begin{aligned}
ax^2 + bx + c = 0 &\Leftrightarrow x^2 + \frac{b}{a}x = -\frac{c}{a} \\
&\Leftrightarrow x^2 + \frac{b}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \\
&\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \\
&\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \\
&\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \\
&\Leftrightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\
&\Leftrightarrow x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
&\Leftrightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
&\Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

□

17. Determine if the following equation defines y as a function of x and explain why or why not.

$$x - 2 = y^2$$

Solution

$$x - 2 = y^2 \Leftrightarrow y = \pm\sqrt{x - 2}$$

Since there are two outputs for y it is $\boxed{\text{not a function}}$

□

18. (a) Using transformations of graphs, graph the function

$$f(x) = -(x+1)^2 + 2$$

(b) On what interval is $f(x)$ increasing?

(c) On what interval is $f(x)$ decreasing?

Solution

(a) Use a graphing utility to check your graph. It is the graph of x^2 reflected across the x-axis, shifted left 1, and shifted up 2

(b) $\boxed{(-\infty, -1)}$

(c) $\boxed{(-1, \infty)}$

□

19. Consider $f(x) = x^8 + x^4$.
Using the definition; determine if the function is odd, even or neither.

Solution

$$\begin{aligned} f(-x) &= (-x)^8 + (-x)^4 \\ &= x^8 + x^4 \\ &= f(x) \end{aligned}$$

So the function is $\boxed{\text{even}}$

□

20. Consider $f(x) = 2x\sqrt{1-x^2}$.
Using the definition; determine if the function is odd, even or neither.

Solution

$$\begin{aligned} f(-x) &= 2(-x)\sqrt{1-(-x)^2} \\ &= -2x\sqrt{1-x^2} \\ &= -f(x) \end{aligned}$$

So the function is $\boxed{\text{odd}}$

□

21. Evaluate the piecewise function at the given values of the independent variable.

$$f(x) = \begin{cases} x + 4, & \text{if } x \geq -4 \\ -(x + 4), & \text{if } x < -4 \end{cases}$$

(a) $f(0) =$

(b) $f(-7) =$

(c) $f(1) =$

(d) Graph

Solution

(a)

$$f(0) = 0 + 4 = \boxed{4}$$

(b)

$$f(-7) = -(-7 + 4) = \boxed{3}$$

(c)

$$f(1) = 1 + 4 = \boxed{5}$$

(d) Use a graphing utility to check your answer.

□

22. Consider the line through the points (-5, -2) and (5,14).

(a) Find the slope of the line.
(b) Find the point-slope equation of the line.
(c) Find the slope-intercept form of the equation of the line.

Solution

(a)

$$m = \frac{14-(-2)}{5-(-5)} = \frac{16}{10} = \boxed{\frac{8}{5}}$$

(b)

$$y-(-2) = \frac{8}{5}(x-(-5)) \text{ or } \boxed{y+2 = \frac{8}{5}(x+5)}$$

(c)

$$y+2 = \frac{8}{5}(x+5) \Leftrightarrow y+2 = \frac{8}{5}x+8 \Leftrightarrow \boxed{y = \frac{8}{5}x+6}$$

□

23. Consider the line passing through the point (3,-3) which is <u>perpendicular</u> to the line $y = \frac{1}{2}x+1$.

(a) Find the slope of the line.
(b) Find the point-slope equation of the line.
(c) Find the slope-intercept form of the equation of the line.

Solution

(a) $\boxed{-2}$

(b)

$$\boxed{y-(-3) = -2(x-3) \text{ or } y+3 = -2(x-3)}$$

(c)

$$y+3 = -2(x-3) \Leftrightarrow y+3 = -2x+6 \Leftrightarrow \boxed{y = -2x+3}$$

□

24. Using transformations of graphs, graph the function $f(x) = -\sqrt{x+3}$.

Solution Use a graphing utility to check your answer. It is the graph of $\sqrt{x}$ reflected across the x-axis and shifted left 3.

□

25. Let $f(x) = x - 8$ and let $g(x) = 4x^2$.

(a) i. Find $(f+g)(x)$ (simplify your answer)

ii. What is the domain of $(f+g)(x)$ in interval notation?

(b) i. Find $(fg)(x)$ (simplify your answer)

ii. What is the domain of $(fg)(x)$ in interval notation?

(c) i. Find $\left(\frac{f}{g}\right)(x)$ (simplify your answer)

ii. What is the domain of $\left(\frac{f}{g}\right)(x)$ in interval notation?

Solution

(a) i.

$$(x-8) + (4x^2) = \boxed{4x^2 + x - 8}$$

ii. $\boxed{(-\infty, \infty)}$

(b) i.

$$(x-8)(4x^2) = \boxed{4x^3 - 32x^2}$$

ii. $\boxed{(-\infty, \infty)}$

(c) i.

$$\boxed{\frac{x-8}{4x^2}}$$

ii. $\boxed{(-\infty, 0) \cup (0, \infty)}$

□

26. Let $f(x) = -3x^2$ and let $g(x) = x + 6$.

(a) Find $(f \circ g)(x)$ (simplify your answer)

(b) What is the domain of $(f \circ g)(x)$ in interval notation?

(c) Find $(g \circ f)(x)$ (simplify your answer)

(d) What is the domain of $(g \circ f)(x)$ in interval notation?

Solution

(a)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x+6) \\ &= -3(x+6)^2 \\ &= -3(x^2 + 12x + 36) \\ &= \boxed{-3x^2 - 36x - 108}\end{aligned}$$

(b) $\boxed{(-\infty, \infty)}$

(c)

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(-3x^2) \\ &= \boxed{-3x^2 + 6}\end{aligned}$$

(d) $\boxed{(-\infty, \infty)}$

$\square$

27. The function $f(x) = \dfrac{9}{x} + 6$ is one-to-one.
Find $f^{-1}(x)$.

Check!

(a) $(f^{-1} \circ f)(x)$
(b) $(f \circ f^{-1})(x)$

Solution

$$\begin{aligned} f(x) = \frac{9}{x} + 6 \to y &= \frac{9}{x} + 6 \\ \to x &= \frac{9}{y} + 6 \\ \to x - 6 &= \frac{9}{y} \\ \to y(x-6) &= 9 \\ \to y &= \frac{9}{x-6} \\ \to & \boxed{f^{-1}(x) = \frac{9}{x-6}} \end{aligned}$$

(a)

$$\begin{aligned} (f^{-1} \circ f)(x) &= f^{-1}(f(x)) \\ &= f^{-1}\left(\frac{9}{x} + 6\right) \\ &= \frac{9}{\frac{9}{x} + \cancel{6} - \cancel{6}} \\ &= \frac{9}{\frac{9}{x}} \\ &= \cancel{9} \cdot \frac{x}{\cancel{9}} \\ &= x \checkmark \end{aligned}$$

(b)

$$\begin{aligned}(f \circ f^{-1})(x) &= f(f^{-1}(x)) \\ &= f\left(\frac{9}{x-6}\right) \\ &= \frac{9}{\frac{9}{x-6}} + 6 \\ &= \cancel{9} \cdot \frac{x-6}{\cancel{9}} + 6 \\ &= x - \cancel{6} + \cancel{6} \\ &= x \checkmark\end{aligned}$$

□

28. Let $f(x) = x^2 - 6,\ x \geq 0$.

(a) Find $f^{-1}(x)$

(b) Graph f and f^{-1}.

(c) Using the graph determine:

i. Domain of f = Range of f^{-1} = (use interval notation)

ii. Domain of f^{-1} = Range of f = (use interval notation)

Solution

(a)

$$\begin{aligned}f(x) = x^2 - 6,\ x \geq 0 &\to y = x^2 - 6,\ x \geq 0 \\ &\to x = y^2 - 6,\ y \geq 0 \\ &\to y^2 = x + 6,\ y \geq 0 \\ &\to y = \pm\sqrt{x+6},\ y \geq 0 \\ &\to f^{-1}(x) = \sqrt{x+6}\end{aligned}$$

(b) Use a graphing utility to check your answer.

(c) i. $[0, \infty)$

ii. $[-6, \infty)$

□

29. Consider the general equation of a circle $x^2 + y^2 + 16x - 2y + 64 = 0$.

(a) Write the equation of the circle in standard form.

(b) What is the radius of the circle?

(c) Give the point that represents the center of the circle.

Solution

(a)

$$\begin{aligned}
&x^2 + y^2 + 16x - 2y + 64 = 0 \\
&\Leftrightarrow (x^2 + 16x) + (y^2 - 2y) = -64 \\
&\Leftrightarrow \left(x^2 + 16x + \left(\frac{16}{2}\right)^2\right) + \left(y^2 - 2y + \left(\frac{-2}{2}\right)^2\right) = -64 + \left(\frac{16}{2}\right)^2 + \left(\frac{-2}{2}\right)^2 \\
&\Leftrightarrow (x^2 + 16x + 64) + (y^2 - 2y + 1) = -64 + 64 + 1 \\
&\Leftrightarrow \boxed{(x+8)^2 + (y-1)^2 = 1}
\end{aligned}$$

(b) $\sqrt{1} = \boxed{1}$

(c) $\boxed{(-8, 1)}$

□

30. Consider the line segment connecting the points (1,4) and (-3,2).
If this line segment is the diameter of a circle, what is the standard equation of the circle?

Solution

$$\begin{aligned} r &= \frac{1}{2}d \\ &= \frac{1}{2}\sqrt{(-3-1)^2 + (2-4)^2} \\ &= \frac{1}{2}\sqrt{16+4} \\ &= \frac{1}{2}\sqrt{4(5)} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{center} &= \text{midpoint} \\ &= \left(\frac{1-3}{2}, \frac{4+2}{2}\right) \\ &= (-1, 3) \end{aligned}$$

$$\boxed{(x+1)^2 + (y-3)^2 = 5}$$

□

31. A person commutes to work a distance of 35 miles and returns on the same route at the end of the day. Their average rate on the return trip is 10 miles per hour faster than their average rate on the outgoing trip.
Create a function $T(x)$ where x represents the rate of the persons trip to work and the output of the function $T(x)$ is the total time spent on the outgoing and return trips in hours.
Hint: Time traveled $= \dfrac{\text{Distance traveled}}{\text{Rate of travel}}$.

Solution

$$\begin{aligned} T(x) &= \frac{35}{x} + \frac{35}{x+10} \\ &= \frac{35(x+10)}{x(x+10)} + \frac{35x}{(x+10)x} \\ &= \frac{35x + 350 + 35x}{x(x+10)} \\ &= \boxed{\frac{70x+350}{x(x+10)}} \end{aligned}$$

$\square$

32. Consider $f(x) = 3x^4 - 5x^3 - 4x^2 - 8x + 5$ by $g(x) = x - 3$ Use *long division* to divide $f(x)$ by $g(x)$. Your answer should be in the form $\dfrac{f(x)}{g(x)} = q(x) + \dfrac{r(x)}{g(x)}$.

Solution

$$\begin{array}{r|l} & \quad 3x^3 \quad + 4x^2 \quad + 8x + 16 \\ \hline x - 3 & \quad 3x^4 - 5x^3 \quad - 4x^2 \quad - 8x \quad + 5 \\ & -3x^4 + 9x^3 \\ & \qquad 4x^3 \quad - 4x^2 \\ & \quad -4x^3 + 12x^2 \\ & \qquad\qquad 8x^2 \quad - 8x \\ & \qquad\quad -8x^2 + 24x \\ & \qquad\qquad\qquad 16x \quad + 5 \\ & \qquad\qquad\quad -16x + 48 \\ & \qquad\qquad\qquad\qquad 53 \end{array}$$

$$\boxed{\frac{3x^4 - 5x^3 - 4x^2 - 8x + 5}{x-3} = 3x^3 + 4x^2 + 8x + 16 + \frac{53}{x-3}}$$

$\square$

33. Let $f(x) = x^3 - 4x^2 - 7x + 10$.

(a) Use *Descartes' rule of signs* to determine the possible number of positive real roots and negative real roots of the polynomial $f(x)$.

(b) Apply the *rational root test* to list all possible rational roots of $f(x)$.

(c) Use either long division or synthetic division to find one root of $f(x)$ from your list in part (b).

Solution

(a) $f(x)$ has 2 sign changes so there are either 2 or zero positive real roots.
$f(-x) = -x^3 - 4x^2 + 7x + 10$ has 1 sign changes so there is 1 negative real root.

(b) Possible numerators: $\pm1, \pm2, \pm5, \pm10$
Possible denominators: ±1
Possible rational roots: $\boxed{\pm1, \pm2, \pm5, \pm10}$

(c)

$$\begin{array}{r|rrrr} & 1 & -4 & -7 & 10 \\ 1 & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

□

34. Apply the Remainder Theorem (with synthetic division) to $f(x) = 2x^3 - 7x + 7$ in order to find $f(3)$.

Solution

$$\begin{array}{r|rrrr} & 2 & 0 & -7 & 7 \\ 3 & & 6 & 18 & 33 \\ \hline & 2 & 6 & 11 & 40 \end{array}$$

So $\boxed{f(3) = 40}$

□

35. Consider the function $f(x) = -3x^2 - 6x + 11$.

(a) Determine whether the function has a minimum or maximum value. Justify your answer.

(b) Find the minimum or maximum value and determine where it occurs.

(c) What is the domain of f?

(d) What is the range of f?

Solution

(a) $\boxed{\text{maximum}}$

(b)

$$\begin{aligned}\text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\\ &= \left(\frac{6}{-6}, f\left(\frac{6}{-6}\right)\right)\\ &= (-1, f(-1))\\ &= (-1, 14)\end{aligned}$$

So the $\boxed{\text{maximum value is 14 at } x = -1}$

(c) $\boxed{(-\infty, \infty)}$

(d) $\boxed{(-\infty, 14]}$

□

36. Consider $f(x) = x^3(4-x)^3(x+5)$

(a) Describe the end behavior of the graph.
A. The graph rises to the left and falls to the right.
B. The graph rises to the left and to the right.
C. The graph falls to the left and rises to the right.
D. The graph falls to the left and to the right.

(b) Find the x-intercepts. State the multiplicity of each intercept and whether the graph crosses the x-axis, or touches the x-axis and turns around, at each intercept.

(c) Find the y-intercept

(d) Determine whether the graph has y-axis symmetry (even function), origin symmetry (odd function), or neither.

(e) Graph

Solution

(a) $\boxed{\text{A}}$

(b)

Zero	Multiplicity	Behavior
0	3	crosses
4	3	crosses
−5	1	crosses

(c) $f(0) = \boxed{0}$

(d)

$$\begin{aligned} f(-x) &= (-x)^3(4-(-x))^3(-x+5) \\ &= -x^3(4+x)^2(5-x) \end{aligned}$$

$\boxed{\text{neither}}$

(e) Use a graphing utility to check your answer.

□

37. Consider $f(x) = \dfrac{15x^2}{3x^2 - 4}$.

(a) Determine whether the graph has y-axis symmetry (even function), origin symmetry (odd function), or neither.

(b) Find the y-intercept (if there is one)

(c) Find the x-intercepts.

(d) Find all the vertical asymptotes.

(e) Find the horizontal asymptote.

(f) Determine one point between and beyond each x-intercept and vertical asymptote, or use multiplicity of zeros and end behavior.

(g) Graph marking all asymptotes.

Solution

(a)

$$\begin{aligned} f(-x) &= \frac{15(-x)^2}{3(-x)^2 - 4} \\ &= \frac{15x^2}{3x^2 - 4} \\ &= f(x) \end{aligned}$$

$\boxed{\text{even}}$

(b) $f(0) = \boxed{0}$

(c) $f(x) = 0 \Leftrightarrow 15x^2 = 0 \Leftrightarrow x = \boxed{0}$

(d)

$$\begin{aligned} 3x^2 - 4 = 0 &\Leftrightarrow 3x^2 = 4 \\ &\Leftrightarrow x^2 = \frac{4}{3} \\ &\Leftrightarrow \boxed{x = \pm\frac{2}{\sqrt{3}}} \end{aligned}$$

(e) $\boxed{y = 5}$

(f) (You'll have to use a calculator for this step)

(g) Use a graphing utility to check your answers.

□

38. A farmer has 240 yards of fencing and wants to enclose a rectangular plot that borders a barn. If the farmer does not fence the side along the barn:

(a) What is the largest area that can be enclosed?

(b) Find the length and width of the plot that will maximize the area.

Solution Let A be the area, l be the length and w be the width. We know that

$$A = lw \text{ and } l + 2w = 240$$

$$l + 2w = 240 \Leftrightarrow l = 240 - 2w \Rightarrow A = w(240 - 2w) = -2w^2 + 240w$$

(a)

$$\begin{aligned} \text{Vertex} &= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \\ &= \left(\frac{-240}{2(-2)}, f\left(\frac{-240}{2(-2)}\right)\right) \\ &= (60, f(60)) \\ &= (60, 7200) \end{aligned}$$

The largest area is $\boxed{7200 \text{ yards}^2}$

(b)

$$\boxed{w = 60\text{yards and } l = 240 - 2(60) = 120 \text{ yards}}$$

□

39. Consider the function $f(x) = x^4 + 3x^3 + x^2 - 3x - 2$. Using that -1 is a root, write f as a product of linear factors.

Solution

$$\begin{array}{r|rrrrr} & 1 & 3 & 1 & -3 & -2 \\ -1 & & -1 & -2 & 1 & 2 \\ \hline & 1 & 2 & -1 & -2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x+1)(x^3 + 2x^2 - x - 2) \\ &= (x+1)(x^2(x+2) - (x+2)) \\ &= (x+1)(x+2)(x^2-1) \\ &= (x+1)(x+2)(x+1)(x-1) \\ &= \boxed{(x+1)^2(x+2)(x-1)} \end{aligned}$$

□