Math 203 Final Exam Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, and homework problems.

(1) Find the following limits

(a) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-2x}$

(b) $\lim_{x\to 7} \frac{x-7}{\sqrt{x}-\sqrt{7}}$

(2) Using the limit definition of a derivative to find the equation of the line tangent to $f(x) = \frac{1}{2x+1}$ when $x = 0$.

(3) Let $f(x) = x^3 + 3x^2$

(a) Find the (open) intervals where f is increasing and where f is decreasing.

(b) Find all relative extrema (both x and y coordinates). Indicate whether it is a relative maximum or relative minimum.

(c) Find the (open) intervals where f is concave up and where f is concave down

(d) Find all inflection point(s) (both x and y coordinates)

(e) Using the information from parts (a)-(d), graph the function. Label all relative extrema and inflection point(s).

(4) Differentiate the following functions. **You do not have to simplify your answer.**

(a) $y = \ln(3x) + e^{4x}$

(b) $f(x) = (3x+4)^3(2x+1)$

(c) $\dfrac{(3x+1)^2}{x^4+5x+6}$

(d) $xe^x + \sqrt{x} + \dfrac{1}{x}$

(5) A clothing store sells a total of 1000 shirts each year. Storage costs \$4 per shirt (based on average number of shirts). To make a new order of shirts costs \$20. Let x represent the number of shirts in each order and r represent the number of orders.

(a) Find a function for the total inventory cost (carrying cost+ordering cost) associated with ordering and storing the shirts.

(b) Find the amount of shirts per order you should make to minimize the inventory cost. How many orders per year is that?

(6) Suppose you want to find the optimal price to sell hamburgers at. When you sell hamburgers at the price of $4 each, you sell 1000. When the price is increased to $5 you sell 800.

(a) Assuming a linear demand curve find the demand equation $p(x)$ relating the hamburger price p to the amount of hamburgers sold x. **You do not have to simplify.** (Hint: Use the above information to get two points that are on the line.)

(b) Use your answer from (a) to find the equation for the revenue produced from selling x hamburgers. **Once again, you do not have to simplify.**

(7) (a) Find $\dfrac{dy}{dx}$ at the point $(0,1)$ for $x^2 + y^2 + 2xy + y = 2$

(b) Use your answer from (a) to find the equation of the line tangent to the curve at $(0,1)$.

(8) Evaluate the following integrals. **Don't forget the $+C$ when necessary!**

(a) $\displaystyle\int_1^2 \left(3x^2 - 4x + \frac{2}{x^2}\right)\, dx$

(b) $\displaystyle\int 4e^{-x}\, dx$

(c) $\displaystyle\int \left(x + 7x^4 + \frac{5}{x} + x^{2/3}\right)\, dx$

(d) $\displaystyle\int_0^{\ln 2} (e^x - e^{-x})\, dx$ (Recall: $e^{\ln a} = a$ for $a > 0$ and $e^0 = 1$)

(9) Let $f(x) = x^3 - 3x$

(a) Find the (open) interval(s) where $f(x)$ is increasing and where it is decreasing.

(b) Find the local maximum and minimum values (both x and y values).

(c) Find the (open) interval(s) where $f(x)$ is concave up and where it is concave down.

(d) Find the inflection point(s) of the function (both x and y values).

(e) Use the information from the previous page to sketch a graph of the function.

(10) Use logarithmic differentiation to find the derivative of $y = \dfrac{(x+1)^3(2x+4)^2}{(3x^2+5)^4}$. **You do not need to simplify your answer.**

(11) Set up **but do not evaluate** the integral(s) you would use to find the area between the curves $y = 12x$ and $y = 3x^3$ from $x = -2$ to $x = 2$

(12) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for each function $f(x,y)$.

(a) $f(x,y) = 3x^2 + 2y^2 - 6xy$

(b) $f(x) = x^2y + xy^2 - \frac{1}{x}$

(c) $h(x) = e^{xy}$

(d) $g(x) = \ln(x^2 + y^2)$

(13) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$ for each function $f(x,y,z)$.

(a) $f(x,y,z) = x^2y + x^2z^3 - \sqrt{x^3y^5z}$

(b) $g(x,y,x) = x^2y^3z^4$

(c) $h(x,y,z) = x^2z + \sqrt{xy}$

(d) $j(x,y,z) = e^{x^2+y^2+z^2}$

(14) Find the critical numbers for the given function of two variables and determine if they are maxima, minima, neither, or undetermined.

(a) $f(x,y) = x^2 + y^2$

(b) $f(x,y) = x^2 + y^2 - 2x - 4y + 2$

(c) $f(x,y) = e^{-x^2-y^2}$

(d) $f(x,y) = 2x^2 - x^4 - y^2$