Homework 10

section 6.2

10) Evaluate $\int_{-2}^{2} \frac{2}{e^{2t}}\,dt$

$$\int_{-2}^{2} \frac{2}{e^{2t}}\,dt = \int_{-2}^{2} 2e^{-2t}\,dt = 2\left(\frac{e^{-2t}}{-2}\right)\Big|_{-2}^{2} = -e^{-2t}\Big|_{-2}^{2} = -e^{-2(2)} - (-e^{-2(-2)})$$

$$= \boxed{-e^{-4}+e^{4}}$$

12) Evaluate $\int_{-2}^{-1} \frac{1+x}{x}\,dx$

$$\int_{-2}^{-1} \frac{1+x}{x}\,dx = \int_{-2}^{-1}\left(\frac{1}{x}+\frac{x}{x}\right)dx = \int_{-2}^{-1}\left(\frac{1}{x}+1\right)dx = \ln|x| + x\,\Big|_{-2}^{-1} = (\ln|-1| + (-1)) - (\ln|-2| + (-2))$$

$$= \ln 1 - 1 - \ln 2 + 2 = 0 - 1 - \ln 2 + 2 = \boxed{1-\ln 2}$$

28) Evaluate $\int_0^3 f(x)\,dx$

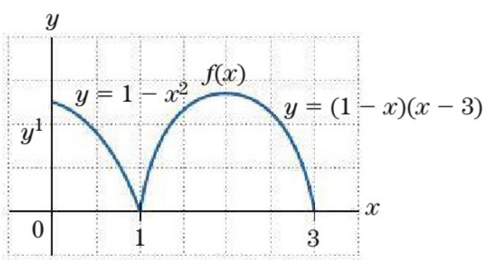


Figure 5

$$\int_0^3 f(x)\,dx = \int_0^1 (1-x^2)\,dx + \int_1^3 (1-x)(x-3)\,dx = \int_0^1 (1-x^2)\,dx + \int_1^3 (x-3-x^2+3x)\,dx$$

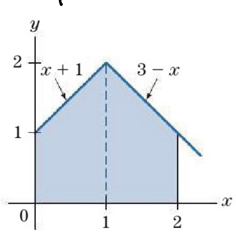
$$= \int_0^1 (1-x^2)\,dx + \int_1^3 (-x^2+4x-3)\,dx$$

$$= \left(x - \frac{x^3}{3}\right)\Big|_0^1 + \left(\frac{-x^3}{3} + \frac{4x^2}{2} - 3x\right)\Big|_1^3 = \left(1-\frac{1}{3}\right) - (0-0) + \left(\frac{-3^3}{3} + \frac{4(3)^2}{2} - 3(3)\right) - \left(\frac{-1}{3} + \frac{4}{2} - 3\right)$$

$$= 1 - \frac{1}{3} + \frac{-27}{3} + \frac{36}{2} - 9 - \left(\frac{-1}{3} + 2 - 3\right) = 1 - \frac{1}{3} - 9 + 18 - 9 + \frac{1}{3} - 2 + 3 = 4 - 2 = \boxed{2}$$

section 6.3

12) Set up the definite integral that gives the area of the shaded region.



$$\int_0^1 (x+1)\,dx + \int_1^2 (3-x)\,dx$$

Section 6.4

8) Find the area of the region between the curve and the x-axis: $f(x) = x(x^2-1)$ from -1 to 1

$x(x^2-1)=0 \Rightarrow x(x-1)(x+1)=0 \Rightarrow x = 0, 1, -1$ split at $x=0$

$f\left(\frac{-1}{2}\right) = \frac{-1}{2}\left(\frac{1}{4}-1\right) = \frac{-1}{2}\left(\frac{-3}{4}\right) = \frac{3}{8} > 0 \qquad f\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{4}-1\right) = \frac{1}{2}\left(\frac{-3}{4}\right) = -\frac{3}{8} < 0$

$x(x^2-1) > 0$ on $[-1, 0]$, $0 > x(x^2-1)$ on $[0, 1]$

$$A = \int_{-1}^{0} x(x^2-1)\,dx - \int_0^1 x(x^2-1)\,dx = \int_{-1}^{0}(x^3-x)\,dx - \int_0^1 (x^3-x)\,dx = \left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Big|_{-1}^{0} - \left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Big|_0^1$$

$$= \left[(0-0)-\left(\frac{1}{4}-\frac{1}{2}\right)\right] - \left[\left(\frac{1}{4}-\frac{1}{2}\right)-(0-0)\right] = \frac{-1}{4}+\frac{1}{2}-\frac{1}{4}+\frac{1}{2} = \frac{-1}{2}+1 = \boxed{\frac{1}{2}}$$

10) Find the area of the region between the curve and the x-axis: $f(x) = x^2+6x+5$ from 0 to 1

$x^2+6x+5=0 \Rightarrow (x+5)(x+1)=0 \Rightarrow x = -1, -5$ both not in $[0,1]$ so no split

test: $f(1/2) = 1/4 + 3 + 5 > 0$

$x^2+6x+5 > 0$ on $[0, 1]$

$$A = \int_0^1 (x^2+6x+5)\,dx = \frac{x^3}{3}+\frac{6x^2}{2}+5x\Big|_0^1 = \frac{x^3}{3}+3x^2+5x\Big|_0^1 = \left(\frac{1}{3}+3+5\right)-(0+0+0) = \frac{1}{3}+8$$

$$= \frac{1}{3}+\frac{24}{3} = \boxed{\frac{25}{3}}$$

Section 9.3

2) Evaluate $\displaystyle\int_2^6 \frac{1}{\sqrt{4x+1}}\,dx$ *

$u = 4x+1 \Rightarrow du = 4dx \Rightarrow dx = \frac{du}{4}$ when $x=2 \Rightarrow u = 8+1 = 9$, $x=6 \Rightarrow u = 24+1 = 25$

$$\int_2^6 \frac{1}{\sqrt{4x+1}}\,dx = \int_9^{25} \frac{1}{\sqrt{u}}\,\frac{du}{4} = \frac{1}{4}\int_9^{25} u^{-1/2}\,du = \frac{1}{4}\left(\frac{u^{1/2}}{1/2}\right)\Big|_9^{25} = \frac{1}{4}(2\sqrt{u})\Big|_9^{25} = \frac{\sqrt{u}}{2}\Big|_9^{25} = \frac{5}{2}-\frac{3}{2} = \boxed{1}$$

8) Evaluate $\displaystyle\int_0^1 \frac{1}{(1+2x)^4}\,dx$

$u = 1+2x \Rightarrow du = 2dx \Rightarrow dx = \frac{du}{2}$ when $x=0 \Rightarrow u = 1+0 = 1$, $x=1 \Rightarrow u = 1+2 = 3$

$$\int_0^1 \frac{1}{(1+2x)^4}\,dx = \int_1^3 \frac{1}{u^4}\,\frac{du}{2} = \frac{1}{2}\int_1^3 u^{-4}\,du = \frac{1}{2}\left(\frac{u^{-3}}{-3}\right)\Big|_1^3 = \frac{-1}{2u^3}\Big|_1^3 = \frac{-1}{54}-\left(-\frac{1}{2}\right) = \frac{-1}{54}+\frac{27}{54} = \frac{26}{54} = \boxed{\frac{13}{27}}$$

classwork graded: CW16 # 1b, 1d   CW 17 # 1b, 3a