Homework 11

Section 6.4

14) Find the area of the region between the curves: $y=x^2+1$ and $y=-x^2-1$ from $x=-1$ to $x=1$

$x^2+1=-x^2-1 \Rightarrow 2x^2+2=0 \Rightarrow 2(x^2+1)=0$ no solution so no split

test $x=0$: $0^2+1=1$

$-0^2-1=-1$

$$\begin{array}{|c|} \hline x^2+1 > -x^2-1 \\ \hline \end{array}$$
$-1 \qquad \qquad 1$

$A = \int_{-1}^{1} (x^2+1)-(-x^2-1)\,dx = \int_{-1}^{1}(2x^2+2)\,dx = \frac{2x^3}{3}+2x \Big|_{-1}^{1} = \left(\frac{2}{3}+2\right)-\left(\frac{-2}{3}-2\right)$ ✳

$= \frac{2}{3}+2+\frac{2}{3}+2 = \frac{4}{3}+4 = \frac{4}{3}+\frac{12}{3} = \boxed{\frac{16}{3}}$

28) Find the area of the region between $y=x^2$ and $y=\frac{1}{x^2}$
   a) From $x=1$ to $x=4$
   b) From $x=\frac{1}{2}$ to $x=4$

$x^2 = \frac{1}{x^2} \Rightarrow x^4=1 \Rightarrow x=\pm 1$

a) no split

$$\begin{array}{|c|} \hline x^2 > \frac{1}{x^2} \\ \hline \end{array}$$
$1 \qquad \qquad 4$

$A = \int_{1}^{4}\left(x^2-\frac{1}{x^2}\right)dx = \int_{1}^{4}(x^2-x^{-2})\,dx = \left(\frac{x^3}{3}+\frac{1}{x}\right)\Big|_{1}^{4} = \left(\frac{64}{3}+\frac{1}{4}\right)-\left(\frac{1}{3}+1\right) = \frac{64}{3}+\frac{1}{4}-\frac{1}{3}-1$

$= \frac{256}{12}+\frac{3}{12}-\frac{4}{12}-\frac{12}{12} = \boxed{\frac{243}{12}}$

b)
$$\begin{array}{|c|c|} \hline \frac{1}{x^2}>x^2 & x^2>\frac{1}{x^2} \\ \hline \end{array}$$
$\frac{1}{2} \qquad 1 \qquad 4$

$A = \int_{1/2}^{1}\left(\frac{1}{x^2}-x^2\right)dx + \underbrace{\int_{1}^{4}\left(x^2-\frac{1}{x^2}\right)dx}_{243/12} = \int_{1/2}^{1}\left(x^{-2}-x^2\right)dx + \frac{243}{12}$

$\int_{1/2}^{1}(x^{-2}-x^2)\,dx = \left(\frac{-1}{x}-\frac{x^3}{3}\right)\Big|_{1/2}^{1} = \left(-1-\frac{1}{3}\right)-\left(-2-\frac{1}{24}\right) = -1-\frac{1}{3}+2+\frac{1}{24} = \frac{-24}{24}-\frac{8}{24}+\frac{48}{24}+\frac{1}{24} = \frac{17}{24}$

$A = \frac{17}{24}+\frac{243}{12} = \frac{17}{24}+\frac{486}{24} = \boxed{\frac{503}{24}}$

classwork graded: 1b, 2a