Homework 12

Section 6.1

62) A soap manufacturer estimates that its marginal cost of producing soap powder is $C'(x) = .2x + 100$ dollars per ton at a production level of x tons per day. Fixed costs are $200 per day. Find the cost of producing x tons of soap powder per day

$C'(x) = 0.2x + 100$, $C(0) = 200$

$C(x) = \int (0.2x + 100) dx = 0.2 \frac{x^2}{2} + 100x + c = 0.1x^2 + 100x + c$ ✳

$C(0) = 0.1(0)^2 + 100(0) + c = c \Rightarrow c = 200$

so $\boxed{C(x) = 0.1x^2 + 100x + 200}$

Section 6.2

36) A company's marginal cost function is given by $C'(x) = 32 + \frac{x}{20}$, where x denotes the number of items produced in 1 day and $C(x)$ is in thousands of dollars. Determine the increase in cost if the company goes from a production level of 15 to 20 items per day

$C'(x) = 32 + \frac{x}{20}$ want $C(20) - C(15)$

$C(20) - C(15) = \int_{15}^{20} \left(32 + \frac{x}{20}\right) dx = 32x + \frac{1}{20}\frac{x^2}{2} \Big|_{15}^{20} = 32x + \frac{x^2}{40} \Big|_{15}^{20}$

$= \left(32(20) + \frac{(20)^2}{40}\right) - \left(32(15) + \frac{(15)^2}{40}\right) \approx \boxed{\$164.38}$

Section 6.5

12) Find the consumer surplus for each of the following demand curves at the given sales level x:

$p = \frac{x^2}{200} - x + 50; \quad x = 20$

$\begin{aligned}\text{consumer} \\ \text{surplus}\end{aligned} = \int_0^{20}\left[\left(\frac{x^2}{200} - x + 50\right) - \left(\frac{(20)^2}{200} - 20 + 50\right)\right] dx = \int_0^{20}\left(\frac{x^2}{200} - x + 50 - 2 + 20 - 50\right) dx$

$= \int_0^{20}\left(\frac{x^2}{200} - x + 18\right) dx = \frac{x^3}{600} - \frac{x^2}{2} + 18x \Big|_0^{20} = \frac{(20)^3}{600} - \frac{(20)^2}{2} + 18(20) \approx \boxed{\$173.33}$

22) Suppose that money is deposited daily in a savings account at an annual rate of $2000. If the account pays 6% interest compounded continuously, approximately how much will be in the account at the end of 2 years?

$\int_0^2 2000 e^{0.06(2-t)} dt = \int_0^2 2000 e^{0.12} e^{-0.06t} dt = 2000 e^{0.12} \int_0^2 e^{-0.06t} dt$ ✳

$= 2000 e^{0.12} \left(\frac{e^{-0.06t}}{-0.06} \Big|_0^2\right) = 2000 e^{0.12}\left(\frac{e^{-0.12}}{-0.06} - \frac{1}{-0.06}\right) = \boxed{\$4249.90}$

<u>Section 7.1</u>

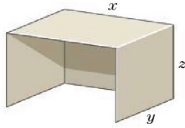2) Let $g(x,y)=\sqrt{x^2+2y^2}$. Compute $g(1,1)$, $g(0,-1)$, and $g(a,b)$.

$g(x,y)=\sqrt{x^2+2y^2}$

$g(1,1)=\sqrt{(1)^2+2(1)^2}=\sqrt{1+2}=\boxed{\sqrt{3}}$

$g(0,-1)=\sqrt{(0)^2+2(-1)^2}=\sqrt{0+2}=\boxed{\sqrt{2}}$

$g(a,b)=\sqrt{(a)^2+2(b)^2}=\boxed{\sqrt{a^2+2b^2}}$

8) Find a formula $C(x,y,z)$ that gives the cost of material for the rectangular figure with dimensions in feet. Assume that the material for the top costs \$3 per square foot and the material for the back and two sides costs \$5 per square foot.

✳

top: y $\boxed{ \atop x}$
\$3 per sq ft

Sides (two of them): $\boxed{}$ z
\$5 per sq ft ^{y}

back: $\boxed{ \atop x}$ z
\$5 per sq ft

$\boxed{C(x,y,z)=3xy+2(5yz)+5xz}$

10) Let $f(x,y)=10x^{2/5}y^{3/5}$. Show that $f(3a,3b)=3f(a,b)$

$f(x,y)=10x^{2/5}y^{3/5}$

$f(a,b)=10a^{2/5}b^{3/5}$

$f(3a,3b)=10(3a)^{2/5}(3b)^{3/5}=10\cdot 3^{2/5}a^{2/5}\cdot 3^{3/5}b^{3/5}=10\cdot 3^{2/5+3/5}a^{2/5}b^{3/5}=10\cdot 3\,a^{2/5}b^{3/5}$

$=3(10a^{2/5}b^{3/5})=3f(a,b)\ \checkmark$

Classwork graded: Cw 19 #3, Cw 20 #2,3